(2)

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

SOUND PROPAGATION IN THE INHOMOGENEOUS OCEAN

by

Daniel L. Devany

June 1991

Thesis Advisor: Lawrence J. Ziomek

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b RESTRICTIVE MARKINGS

2a SECURITY CLASSIFICATION AUTHORITY

2b DECLASSIFICATION / DOWNGRADING SCHEDULE

3 DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release; distribution is unlimited

4 PERFORMING ORGANIZATION REPORT NUMBER(S)

5 MONITORING ORGANIZATION REPORT NUMBER(S)

6a NAME OF PERFORMING ORGANIZATION
Naval Postgraduate School

6b OFFICE SYMBOL (If applicable)
EC

7a NAME OF MONITORING ORGANIZATION
Naval Postgraduate School

6c ADDRESS (City, State, and ZIP Code)
Monterey, CA 93943-5000

7b ADDRESS (City, State, and ZIP Code)
Monterey, CA 93943-5000

8a NAME OF FUNDING / SPONSORING ORGANIZATION

8b OFFICE SYMBOL (If applicable)

9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c ADDRESS (City, State, and ZIP Code)

10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)
SOUND PROPAGATION IN THE INHOMOGENEOUS OCEAN

12 PERSONAL AUTHOR(S)
DEVANY, Daniel L.

13a TYPE OF REPORT
Master's Thesis

13b TIME COVERED
FROM ________ TO ______

14 DATE OF REPORT (Year, Month, Day)
1991 June

15 PAGE COUNT
61

16 SUPPLEMENTARY NOTATION The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the US Government.

17 COSATI CODES

FIELD	GROUP	SUB-GROUP

18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Underwater Acoustics; Phase Integrals; Ocean Transfer Functions

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

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Two applications of ray acoustics theory are investigated as accurate, efficient, alternatives to direct numerical intergration of the WKB phase integral. Both applications base phase computations on signal travel time. The difference is their treatment of the sound-speed versus depth data pairs. One forms a sound-speed profile by using the piecewise linear approximation method while the other uses an Akima cubic spline fit to the data.

Each method can identify source-to-receiver eigenrays and provide ray trace plots.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS

21 ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a NAME OF RESPONSIBLE INDIVIDUAL
ZIOMEK, Lawrence J.

22b TELEPHONE (Include Area Code)
408-646-3206

22c OFFICE SYMBOL
EC/Zm

DD Form 1473, JUN 86

Previous editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE

S/N 0102-LF-014-6603

UNCLASSIFIED

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Sound Propagation in the Inhomogeneous Ocean

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1991

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ABSTRACT

By using a linear systems theory approach, an ocean medium transfer function based on the WKB approximation can be developed. The phase computations for the transfer function are made by evaluating the WKB phase integral.

Two applications of ray acoustics theory are investigated as accurate, efficient alternatives to direct numerical integration of the WKB phase integral. Both applications base phase computations on signal travel time. The difference is their treatment of the sound-speed versus depth data pairs. One forms a sound-speed profile by using the piecewise linear approximation method while the other uses an Akima cubic spline fit to the data.

Each method can identify source-to-receiver eigenrays and provide ray trace plots.

TABLE OF CONTENTS

I. INTRODUCTION

A. BACKGROUND

In analyzing ocean acoustic pulse-propagation problems, accurate phase calculations must be performed in order to predict the acoustic signal at the receiver. Linear systems theory provides analytical expressions for analyzing the propagating acoustic field [Refs. 1 - 4]. These well-known equations form the basis for the FORTRAN programs developed in this thesis.

Program input is depth versus sound-speed data pairs where the speed of sound is an arbitrary function of depth. The ocean is viewed as a linear, time-variant, space-variant filter. The WKB approximation can specify this filter's transfer function [Refs. 1 - 4]. For the arbitrary sound-speed profile, no exact transfer function exists. The transfer function requires a method for evaluating signal phase at the receiver.

Two phase evaluation methods are presented and contrasted. Each method calculates the phase of the acoustic signal for a specified horizontal range. The first method overlays a piecewise linear profile on the input sample values for the speed of sound at various depths. The theory of ray acoustics is used to calculate travel time and phase. The second method fits a smooth cubic spline curve to the input samples. This method solves the propagation problem using a system of three, first-order differential equations [Ref. 5]. Both methods make phase calculations using signal travel times. Additionally, each method is capable of identifying eigenrays or rays that directly connect the signal source to the receiver.

Another method is introduced to validate results. Direct numerical integration is performed to calculate phase for a single gradient, linear, sound-speed profile. A separate FORTRAN program implements this numerical integration routine providing a totally independent verification.

An overview of the theory behind each method is presented in Chapter II. Computer simulation results are presented in Chapter III for various input sound-speed profiles. Tabular results of phase calculations are presented for each of the three analysis methods. Using input from the piecewise linear and the cubic spline/differential equation solution methods, ray traces of the propagating field are presented as a visual aid in interpreting the results.

II. THEORETICAL BACKGROUND FOR THE EVALUATION OF PHASE INTEGRALS

A. UNDERWATER ACOUSTIC PULSE PROPAGATION

Ocean acoustic pulse-propagation models can be derived by using the principles of linear, time-variant, space-variant, systems theory and the physics of wave propagation in inhomogeneous media. Linear systems theory allows for the development of an ocean medium transfer function. An ocean medium transfer function that is based on the WKB approximation has been derived and is given by References 1 and 2 as follows:

$$H(f, f_r, y_0; y) = \frac{A}{\sqrt{|k_y(y)|}} \, e^{-j\int_{y_0}^{y} k_y(\zeta)\,d\zeta} \, e^{j2\pi f_y (y-y_0)} \tag{2.1}$$

where

- f is the frequency in Hz
- f_r is the radial, spatial frequency in cycles per meter
- y_0 is the source depth in meters

$$A = j\frac{\sqrt{|k_y(y_0)|}}{4\pi f_y} \tag{2.2}$$

$k_y(y)$ is the propagation vector component in the Y direction with units of radians per meter and is given by

$$k_y(y) = \begin{cases} \pm\ 2\pi\{[f/c(y)]^2 - f_r^2\}^{1/2}, & f_r < f/c(y) \quad (2.3) \\ \mp\ j2\pi\{f_r^2 - [f/c(y)]^2\}^{1/2}, & f_r > f/c(y) \quad (2.4) \end{cases}$$

$c(y)$ is the sound-speed expressed as a function of depth with units of meters per second

f_y is the transmitted (input) spatial frequency in the Y direction at the source in cycles per meter and is given by

$$f_Y = \begin{cases} \pm\ [(f/c_0)^2 - f_r^2]^{1/2}, & f_r < f/c_0 \quad (2.5) \\ \mp\ [f_r^2 - (f/c_0)^2]^{1/2}, & f_r > f/c_0 \quad (2.6) \end{cases}$$

and

c_0 is the speed of sound in meters per second at the source depth y_0, that is, $c_0 = c(y_0)$;

The plus (minus) sign in Eqs. (2.3) and (2.5) is chosen whenever $y - y_0 > 0$ $(y - y_0 < 0)$. The minus (plus) sign in Eqs. (2.4) and (2.6) corresponds to the plus (minus) sign in Eqs. (2.3) and (2.5). This thesis deals only with propagating waves. Therefore, Eqs. (2.4) and (2.6) representing the generation of evanescent waves (i.e., decaying exponentials), will not be used.

The evaluation of the phase integral in Eq. (2.1), namely,

$$\theta_y(y) = \int_{y_0}^{y} k_y(\zeta)\,d\zeta \qquad (2.7)$$

has been attempted in past studies in which solution techniques included direct integration and binomial expansions [Ref. 3].

Computer programs using these techniques proved to be very expensive in terms of CPU time. This thesis compares four different methods for evaluating the phase integral. The four methods are presented in the following sections.

B. DIRECT INTEGRATION

Computer code was developed to directly evaluate the phase integral given by (Eq. 2.7) using numerical integration routines from the International Math and Statistics Library (IMSL). Direct integration is a time-consuming technique that was applied only under the following constraints:

- free-space acoustic propagation
- linear sound-speed profile with a single gradient g in inverse seconds and
- propagating signals that have not passed through a turning point in the medium.

Expressions for the propagation vector component $k_y(y)$ of the phase integral are given by Eqs. (2.3) and (2.4). For a linear sound-speed profile with a single gradient, the dependence of c(y) on depth y is given by

$$c(y) = c(y_0) + g(y - y_0) \qquad (2.8)$$

where

$c(y_0)$ is the sound-speed at the source depth y_0 meters g is the sound-speed gradient, and

y is the desired depth for sound-speed.

Since the direct integration approach is limited to free-space, single gradient, propagation problems, it is used only to validate the results of other solution techniques for several simple test cases. The results will be compared against the next two methods to be presented. These methods will apply the theory of ray acoustics [Ref. 4 & 5] to find travel time. Phase calculations will be based on travel time calculations.

C. PHASE CALCULATIONS BASED ON PIECEWISE LINEAR SOUND-SPEED PROFILES

A FORTRAN program developed by Lim [Ref. 5] was the ray acoustics propagation code that was used to draw ray trace plots and to calculate travel time, in addition to the depth, angle of propagation and path length along a ray path as a function of horizontal range. This method applies ray acoustics to a piecewise linear sound-speed model of the ocean medium. Based on the input sound-speed versus depth data pairs, layers are defined in the ocean medium. Each layer has an upper and lower boundary at specific depths. A constant sound-speed gradient g is calculated for each layer. The sound-speed for any desired depth is computed using Eq. (2.8) with the appropriate gradient.

In a constant gradient medium, ray acoustics theory allows calculation of travel time with closed-form equations. For an incremental increase in horizontal range from the source, the angle of arrival for the propagating ray can be shown to be [Ref. 5].

$$\beta(y) = \cos^{-1}[\cos\beta_0 - b*g(y) * rngstp] \qquad (2.9)$$

where

β_0 is the ray launch angle,

$g(y)$ is the gradient in the layer at depth y, and

rngstp is the incremental increase in horizontal range.

The ray parameter b in Eq. (2.9) is given by [Refs. 4 & 5].

$$b = \sin\beta_0 \,/\, c(y_0) \qquad (2.10)$$

Knowing the arrival angle $\beta(y)$, the ray depth y is given by [Refs. 4 & 5]

$$y = y_0 + \frac{c_0}{g}\left[\frac{\sin\beta(y)}{\sin\beta(y_0)} - 1\right] \qquad (2.11)$$

The arrival angle also allows calculation of travel time from [Refs. 4 & 5]

$$\tau = \frac{1}{g}\ln\left[\frac{\tan[\beta(y)/2]}{\tan[\beta(y_0)/2]}\right] \qquad (2.12)$$

Once the travel time (in seconds) is found for the desired horizontal range, total phase is given by

$$\Theta_T = 2\pi f\tau \qquad (2.13)$$

The phase integral given by Eq. (2.7) represents a phase change in the depth (y) direction. The constant value of the propagation vector component in the radial direction allows for easy calculation of phase change in the radial direction. This value is given by

$$\theta_R = 2\pi f_r * hrzrng \tag{2.14}$$

where hrzrng is the total horizontal range traveled. The phase in the depth direction is

$$\theta_y = \theta_T - \theta_R . \tag{2.15}$$

Phase expressed in radians is a modulo 2π function. The proper solution for the phase integral Eq. (2.15) must be expressed as a modulo 2π function:

$$\theta_y = \text{modulo}\ (\theta_y\ ,\ 2\pi\ )\ . \tag{2.16}$$

This method (referred to as method 1 in Ref. 5) is capable of analyzing propagating rays at any horizontal range in either free space or a bounded medium.

D. PHASE CALCULATIONS BASED ON AKIMA CUBIC SPLINES AND ORDINARY DIFFERENTIAL EQUATIONS

This application of ray acoustics applies a more sophisticated treatment to the depth versus sound-speed data pairs sampled from the ocean medium. The data pairs are used to form a smooth Akima cubic spline sound-speed profile. Splines offer the advantage of using all data points in generating a profile, and they place no restrictions on spacing between data points. The Akima version of the cubic spline was chosen for its excellent ability to combat wiggles in the profile, that is, it suppresses oscillations that would cause overshoots and undershoots in the sound-speed versus depth profile.

This method uses ray acoustics theory to generate a system of three, first-order differential equations [Ref. 5]:

$$\dot{y}_1 = y_2 \tag{2.17}$$

$$\dot{y}_2 = \frac{-\dot{c}(y_1)}{b^2 c^3(y_1)} \tag{2.18}$$

and

$$\dot{y}_3 = \frac{1}{b c^2(y_1)} \tag{2.19}$$

where

$c(y_1)$ is the derivative of the sound-speed with respect to depth at depth y_1,

y_1 is the ray depth,

y_2 is the cotangent of the ray's arrival angle $\beta(y)$, and

y_3 is the travel time of the ray.

Once solved, this system of differential equations allows phase to be calculated from travel time. The phase calculation is performed exactly as shown for the piecewise linear sound-speed profile.

Like the previous method, the Akima cubic spline/differential equation solution can be used on free space and bounded media problems.

E. EIGENRAYS

Eigenrays are propagating rays that exactly connect the sound source to the receiver. The FORTRAN propagation code developed can search for and identify eigenrays. The input required is

- depth versus sound-speed data pairs,
- source depth in meters,
- receiver depth and range in meters,
- angle step in degrees between possible eigenrays to be evaluated, and
- allowed depth error denoted y_{error} in meters.

Rays passing within the allowed error or tolerance y_{error} of the receiver are identified as eigenrays. The eigenray mode can employ either the piecewise linear sound-speed profile or the Akima cubic spline/differential equation method for ray propagation.

The eigenrays are found by trial-and-error. This search method was chosen after the IMSL DBVPMS program failed to solve the problem. The DBVPMS program is a differential equation solver that was applied to the system of differential equations given by Eqs. (2.17) through (2.19). The routine uses the shooting method to find eigenray solutions to boundary value problems. It was unable to converge to a solution. Solutions for the acoustic problems investigated are difficult for this algorithm because of

- long propagation distances,

- the inhomogeneous ocean presents a continuously varying medium with discrete boundaries, and

- the system of differential equations has non-constant coefficients whose values are a function of depth.

When the eigenray mode is selected, rays that are not refracted or reflected to within y_{error} of the receiver are discarded.

III. COMPUTER SIMULATION RESULTS

A. OVERVIEW

The computer simulation results presented in this chapter perform phase calculations intended to:

- show that ray acoustics can be used to evaluate the phase integral producing the same results as direct numerical integration,

- show that the two methods of applying ray acoustics theory (presented in Chapter II) produce equivalent results within an acceptable tolerance,

- compare speed versus accuracy for the two methods for a variety of test cases, and

- demonstrate the ability to identify eigenrays using either of the ray acoustics methods.

B. DIRECT INTEGRATION

In the previous chapter, $\theta_y(y)$ was shown as the solution to the phase integral in the depth direction.

$$\theta_y(y) = \int_{y_0}^{y} k_y(\zeta)\, d\zeta \tag{2.7}$$

The straight-forward evaluation technique is the brute force approach using direct numerical integration. The phase integral will be evaluated using direct numerical integration for a simple test case. The results will then be compared to phase values obtained using ray acoustics calculating phase values from travel times.

Conditions for the test case are:

- source depth of 10 meters,
- receiver depth of 100 meters,
- linear sound-speed profile with a single, constant gradient of 0.016 sec^{-1} and
- four values of radial spatial frequency (FR values) will be evaluated.

For each FR value, tabular results will show the corresponding ray launch angle $\beta(y_0)$. The relation between the FR value and this launch angle is [Ref. 4]:

$$\beta(y_0) = \sin^{-1} [FR * c(y_0)/f] \qquad (3.1)$$

where

f is the frequency in HZ, and

$c(y_0)$ is the speed of sound at the source depth.

Table 3.1 shows the results produced by direct numerical integration. Tables 3.2 and 3.3 show the results for the Akima cubic spline/ordinary differential equation solver (ODE solver) and the piecewise linear approximation ray acoustics techniques, respectively.

The parameter of interest is THETAY representing the phase change in the depth or Y direction. Table 3.1 shows these θ values to be approximately 92, 86, 75, and 56 radians for the four FR values chosen. Tables 3.2 and 3.3 contain four sets of entries, each corresponding to an FR value. These entries begin at 0 meters range and end at the range corresponding to 100 meters depth; thus, solving the phase integral for the θ_y

TABLE 3.1
PHASE INTEGRAL SOLUTION BY DIRECT NUMERICAL INTEGRATION

INPUT DATA FOR PHASE INTEGRAL EVALUATION

F = 250.0 HZ   YL = 10.0 M   YU = 100.0 M   RATIO = 0.9999   NFR = 5
EABS = 0.000000   EREL = 0.000100
YREF = 0.0 M   CYREF = 1500.0 M/SEC   G = 0.016000 (1/SEC)

METHOD = 2

EVALUATION BY IMSL10 ROUTINE DQDAG WITH IRULE = 2

FR(CYCLES/M)	BETAY0(DEG)	RANGE AT YU	ANGLE OF ARRIVAL BETAY(DEG)	TURNING POINT DEPTH(M)	TURNING POINT RANGE(M)	THETAR(RAD)	THETAY(RAD)
0.0332944859	11.525	18.36009	11.536	0.37555E+06	0.45984E+06	0.38408E+01	0.9229167936E+02
0.0665889718	23.552	39.25213	23.576	0.14090E+06	0.21510E+06	0.16423E+02	0.8633840249E+02
0.0998834576	36.824	67.43892	36.866	0.62682E+05	0.12522E+06	0.42324E+02	0.7537857191E+02
0.1331779435	53.049	119.80722	53.122	0.23574E+05	0.70527E+05	0.10025E+03	0.5657368413E+02

TABLE 3.2
PHASE INTEGRAL SOLUTION BY ODE SOLVER

ERAYS = F   PRTALL = T
Y0 = 10.0 M   DEPTH = 100.0 M   FREQC = 250.0 HZ   CMAX = 1501.6 M/SEC
FREQC/CMAX = 0.16649 CYCLES/M   RATIO = 0.9999   FRMAX = 0.16647 CYCLES/M   NFRS = 5
DLTFR = 0.033294 CYCLES/M   RNGSTP = 1.0 M   HRZRNG = 150.0 M   YR = 40.0 M
NOTE: FRMIN = DLTFR

FR = 0.332944858817 26E-01 CYCLES/M
BETA0 = 11.52457521170979 DEG

RANGE(M)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	BTEST	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)
0.0	10.00	0.000000	11.525	0.13318E-03	0.0000000E+00	0.0000000E+00	0.0000000E+00
18.4	100.00	0.061200	11.536	0.13318E-03	0.9613253E+02	0.3840846E+01	0.9229168E+02

FR = 0.665889717634 52E-01 CYCLES/M
BETA0 = 23.55170283980132 DEG

RANGE(M)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	BTEST	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)
0.0	10.00	0.000000	23.552	0.26636E-03	0.0000000E+00	0.0000000E+00	0.0000000E+00
39.3	100.00	0.065420	23.576	0.26636E-03	0.1027611E+03	0.1642273E+02	0.8633840E+02

FR = 0.998834576451 78E-01 CYCLES/M
BETA0 = 36.82440911644773 DEG

RANGE(M)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	BTEST	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)
0.0	10.00	0.000000	36.824	0.39953E-03	0.0000000E+00	0.0000000E+00	0.0000000E+00
67.4	100.00	0.074932	36.866	0.39953E-03	0.1177023E+03	0.4232374E+02	0.7537857E+02

FR = 0.133177943526 90E+00 CYCLES/M
BETA0 = 53.04928575821315 DEG

RANGE(M)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	BTEST	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)
0.0	10.00	0.000000	53.049	0.53271E-03	0.0000000E+00	0.0000000E+00	0.0000000E+00
119.8	100.00	0.099829	53.122	0.53271E-03	0.1568262E+03	0.1002525E+03	0.5657368E+02

TABLE 3.3
PHASE INTEGRAL SOLUTION BY PIECEWISE LINEAR APPROXIMATION

```
RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1+L4

NDATA =  4                       NUMBER OF GRADIENTS =  3
G( 1) =  0.16000E-01 1/SEC    G( 2) =  0.16000E-01 1/SEC
G( 3) =  0.16000E-01 1/SEC

ERAYS = F                        PRTALL = T
YO =   10.0 M                    DEPTH =  100.0 M     FREQ =      250.0 HZ       CMAX =  1501.6 M/SEC
FREQ/CMAX = 0.16649 CYCLES/M     RATIO = 0.9999       FRMAX = 0.16647 CYCLES/M   NFRS =  5
DLTFR =  0.033294 CYCLES/M       RNGSTP =  1.0 M      HRZRNG =      150.0 M      YR =   40.0 M
NOTE: FRMIN = DLTFR
```

FR = 0.33294485881726E-01 CYCLES/M
BETAO = 11.52457521170980 DEG

RANGE(M)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	BTEST	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)
0.0	10.00	0.000000	11.525	0.13318E-03	0.0000000E+00	0.0000000E+00	0.0000000E+00
19.4	100.00	0.061200	11.536	0.13318E-03	0.9613253E+02	0.3840846E+01	0.9229168E+02

FR = 0.66588971763452E-01 CYCLES/M
BETAO = 23.55170280980134 DEG

RANGE(M)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	BTEST	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)
0.0	10.00	0.000000	23.552	0.26636E-03	0.0000000E+00	0.0000000E+00	0.0000000E+00
39.3	100.00	0.065420	23.576	0.26636E-03	0.1027611E+03	0.1642273E+02	0.8633840E+02

FR = 0.99883457645178E-01 CYCLES/M
BETAO = 36.82440911644776 DEG

RANGE(M)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	BTEST	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)
0.0	10.00	0.000000	36.824	0.39953E-03	0.0000000E+00	0.0000000E+00	0.0000000E+00
67.4	100.00	0.074932	36.866	0.39953E-03	0.1177023E+03	0.4232374E+02	0.7537857E+02

FR = 0.13317794352690E+00 CYCLES/M
BETAO = 53.04928575821320 DEG

RANGE(M)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	BTEST	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)
0.0	10.00	0.000000	53.049	0.53271E-03	0.0000000E+00	0.0000000E+00	0.0000000E+00
119.8	100.00	0.098839	53.122	0.53271E-03	0.1568262E+03	0.1002525E+03	0.5657368E+02

value when a ray travels from 10 to 100 meters depth. The significance of these three tables is that the ray acoustics methods are yielding the same phase values θ_y as the direct integration method.

The conclusion to be derived from this is that ray acoustics theory can be used to accurately evaluate the phase integral.

C. VARYING SOUND-SPEED GRADIENTS

Both ray acoustics methods will now be applied to three different test cases. A zero gradient, positive gradient (+0.016 sec^{-1}), and a negative gradient (-0.016 sec^{-1}) linear sound-speed profile will be used. These simulations will show that both methods produce equivalent results within an acceptable tolerance for a variety of media.

The conditions common to each simulation run are:

- source depth of 10 meters,
- horizontal range of 1 kilometer,
- ocean depth of 100 meters,
- speed of sound at the surface of 1500 meters per second, and
- five values of radial spatial frequency (FR) evaluated.

The results are presented in Tables 3.4 through 3.9. The phase integral solution is the phase change in the Y (depth) direction listed as the MODULO TWOPI THETAY value in radians.

TABLE 3.4
ZERO GRADIENT, PIECEWISE LINEAR SOLUTION

RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1A
NDATA = 4   NUMBER OF GRADIENTS = 3
G(1) = 0.00000E+00 1/SEC   G(2) = 0.00000E+00 1/SEC
G(3) = 0.00000E+00 1/SEC

Y0 = 10.0 M   DEPTH = 100.0 M   FREQC = 250.0 HZ   CMAX = 1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M   RATIO = 0.9999   FRMAX = 0.16665 CYCLES/M   NFRS = 5
DLTFR = 0.033330 CYCLES/M   RNGSTP = 1.0 M   HRZRNG = 1000.0 M
NOTE: FRMIN = DLTFR

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.536	90.51	3.333667	168.464	0.5236511E+04	0.2094186E+03	0.5027093E+04	0.5445951
2	23.576	98.44	1.666833	156.424	0.2618256E+04	0.4188371E+03	0.2199419E+04	0.3037135
3	36.866	56.46	1.111222	143.134	0.1745504E+04	0.6282557E+03	0.1117248E+04	5.1243041
4	53.122	39.79	0.833417	126.878	0.1309128E+04	0.8376743E+03	0.4714536E+03	0.2146886
5	89.190	24.14	0.666733	89.190	0.1047302E+04	0.1047093E+04	0.2094500E+00	0.2094500

TOTAL CPU TIME = 0 MIN , 22.63 SEC

TABLE 3.5
ZERO GRADIENT, ODE SOLVER SOLUTION

RAY TRACING USING AKIMA CUBIC SPLINE & ODE SOLVER
CASE 2A
NDATA = 4

Y0 = 10.0 M   DEPTH = 100.0 M   FREQC = 250.0 HZ   CMAX = 1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M   RATIO = 0.9999   FRMAX = 0.16665 CYCLES/M   NFRS = 5
DLTFR = 0.033330 CYCLES/M   RNGSTP = 1.0 M   HRZRNG = 1000.0 M
NOTE: FRMIN = DLTFR

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.536	90.51	3.333667	168.464	0.5236511E+04	0.2094186E+03	0.5027093E+04	0.5445951
2	23.576	98.44	1.666833	156.424	0.2618256E+04	0.4188371E+03	0.2199419E+04	0.3037135
3	36.866	56.46	1.111222	143.134	0.1745504E+04	0.6282557E+03	0.1117248E+04	5.1243041
4	53.122	39.79	0.833417	126.878	0.1309128E+04	0.8376743E+03	0.4714536E+03	0.2146886
5	89.190	24.14	0.666733	89.190	0.1047302E+04	0.1047093E+04	0.2094500E+00	0.2094500

TOTAL CPU TIME = 15 MIN , 52.38 SEC

TABLE 3.6
POSITIVE GRADIENT, PIECEWISE LINEAR SOLUTION

RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1B
NDATA = 4 NUMBER OF GRADIENTS = 3
G(1) = 0.16000E-01 1/SEC G(2) = 0.16000E-01 1/SEC
G(3) = 0.16000E-01 1/SEC

YO = 10.0 M DEPTH = 100.0 M FREQC = 250.0 HZ CMAX = 1501.6 M/SEC
FREQC/CMAX = 0.16649 CYCLES/M RATIO = 0.9999 FRMAX = 0.16647 CYCLES/M NFRS = 5
DLTFR = 0.033294 CYCLES/M RNGSTP = 1.0 M HRZRNG = 1000.0 M
NOTE: FRMIN = DLTFR

NFR	BETAO(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.525	87.80	3.333652	168.466	0.5236488E+04	0.2091954E+03	0.5027293E+04	0.7442569
2	23.552	96.99	1.666824	156.425	0.2618241E+04	0.4183908E+03	0.2199850E+04	0.7348999
3	36.824	55.38	1.111192	143.155	0.1745456E+04	0.6275863E+03	0.1117870E+04	5.7462108
4	53.049	38.73	0.833378	126.927	0.1309066E+04	0.8367817E+03	0.4722847E+03	1.0457792
5	87.363	50.71	0.667000	87.974	0.1047721E+04	0.1045977E+04	0.1743798E+01	1.7437978

TOTAL CPU TIME = 0 MIN . 22.05 SEC

TABLE 3.7
POSITIVE GRADIENT, ODE SOLVER SOLUTION

RAY TRACING USING AKIMA CUBIC SPLINE & ODE SOLVER
CASE 2B
NDATA = 4

YO = 10.0 M DEPTH = 100.0 M FREQC = 250.0 HZ CMAX = 1501.6 M/SEC
FREQC/CMAX = 0.16649 CYCLES/M RATIO = 0.9999 FRMAX = 0.16647 CYCLES/M NFRS = 5
DLTFR = 0.033294 CYCLES/M RNGSTP = 1.0 M HRZRNG = 1000.0 M
NOTE: FRMIN = DLTFR

NFR	BETAO(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.525	87.80	3.333652	168.466	0.5236488E+04	0.2091954E+03	0.5027293E+04	0.7442693
2	23.552	96.99	1.666824	156.425	0.2618241E+04	0.4183908E+03	0.2199850E+04	0.7349005
3	36.824	55.38	1.111192	143.155	0.1745456E+04	0.6275863E+03	0.1117870E+04	5.7462125
4	53.049	38.73	0.833378	126.927	0.1309066E+04	0.8367817E+03	0.4722847E+03	1.0457793
5	87.363	50.71	0.667000	87.974	0.1047721E+04	0.1045977E+04	0.1743798E+01	1.7437978

TOTAL CPU TIME = 16 MIN . 24.30 SEC

TABLE 3.8
NEGATIVE GRADIENT, PIECEWISE LINEAR SOLUTION

```
RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1C
NDATA =  4                      NUMBER OF GRADIENTS =  3
G( 1) = -0.16000E-01 1/SEC   G( 2) = -0.16000E-01 1/SEC
G( 3) = -0.16000E-01 1/SEC

Y0 =   10.0 M                  DEPTH =  100.0 M    FREQC = 250.0 HZ           CMAX =  1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M  RATIO = 0.9999      FRMAX = 0.16665 CYCLES/M  NFRS =  5
DLTFR =  0.033330 CYCLES/M     RNGSTP =  1.0 M     HRZRNG =     1000.0 M
NOTE: FRMIN = DLTFR
```

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.535	87.78	3.337240	168.475	0.5242124E+04	0.2094186E+03	0.5032706E+04	6.1576516
2	23.573	96.98	1.668622	156.450	0.2621065E+04	0.4188371E+03	0.2202228E+04	3.1134653
3	36.861	55.32	1.112438	143.160	0.1747414E+04	0.6282557E+03	0.1119159E+04	0.7515713
4	53.114	38.63	0.834345	126.909	0.1310586E+04	0.8376743E+03	0.4729117E+03	1.6727921
5	88.835	35.67	0.667046	88.224	0.1047793E+04	0.1047093E+04	0.6999612E+00	0.6999612

TOTAL CPU TIME = 0 MIN , 22.61 SEC

TABLE 3.9
NEGATIVE GRADIENT, ODE SOLVER SOLUTION

```
RAY TRACING USING AKIMA CUBIC SPLINE & ODE SOLVER
CASE 2C
NDATA =  4

Y0 =   10.0 M                  DEPTH =  100.0 M    FREQC = 250.0 HZ           CMAX =  1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M  RATIO = 0.9999      FRMAX = 0.16665 CYCLES/M  NFRS =  5
DLTFR =  0.033330 CYCLES/M     RNGSTP =  1.0 M     HRZRNG =     1000.0 M
NOTE: FRMIN = DLTFR
```

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.535	87.77	3.337240	168.475	0.5242124E+04	0.2094186E+03	0.5032706E+04	6.1576656
2	23.573	96.98	1.668622	156.450	0.2621065E+04	0.4188371E+03	0.2202228E+04	3.1134683
3	36.861	55.32	1.112438	143.160	0.1747414E+04	0.6282557E+03	0.1119159E+04	0.7515729
4	53.114	38.63	0.834345	126.909	0.1310586E+04	0.8376743E+03	0.4729117E+03	1.6727921
5	88.835	35.67	0.667046	88.224	0.1047793E+04	0.1047093E+04	0.6999612E+00	0.6999612

TOTAL CPU TIME = 16 MIN , 21.26 SEC

Tables 3.4 and 3.5 show the results for the zero gradient case. A comparison between two tables shows that the phase values (MODULO TWOPI THETAY) agree perfectly. Additionally, all other calculated values in the tables agree. The only difference is the TOTAL CPU TIME. Both methods are using a conservative range step (RNGSTP) of 1 meter, i.e., the incremental increase in range is 1 meter in the propagation calculations. For these conditions, the ODE solver is taking over 30 times longer to run the simulation.

Tables 3.6 and 3.7 show the results for the positive gradient case. A comparison between these two tables, as well as the negative gradient results shown in Tables 3.8 and 3.9, shows acceptable agreement between the two very different calculation methods. The only discrepancies in calculated values occurs in the modulo 2π θ_y phase values needed for solving the phase integral. The phase calculation is most challenging because of the nature of phase (being a modulo 2π function). The discrepancies for both the positive and negative gradients are on the order of hundred-thousandths of a radian or less. The CPU times continue to follow the pattern seen earlier that the ODE solver requires over 30 times longer to complete a simulation run.

The following ray trace plots, Figures 3.1 through 3.6, correspond to Tables 3.4 through 3.9. The plots assist interpretation of the data, but as expected from the tabular

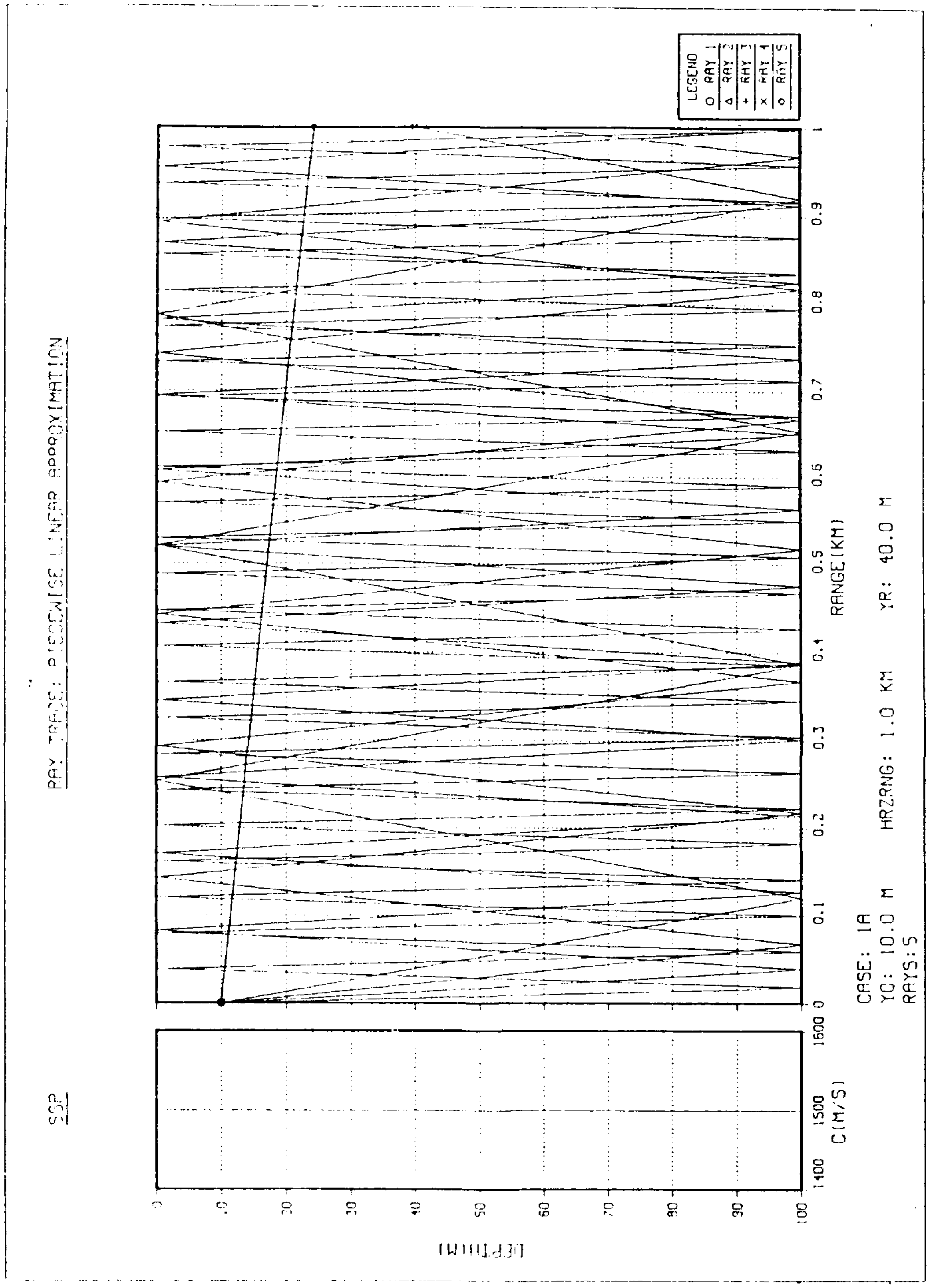

Figure 3.1 Ray trace corresponding to Table 3.4

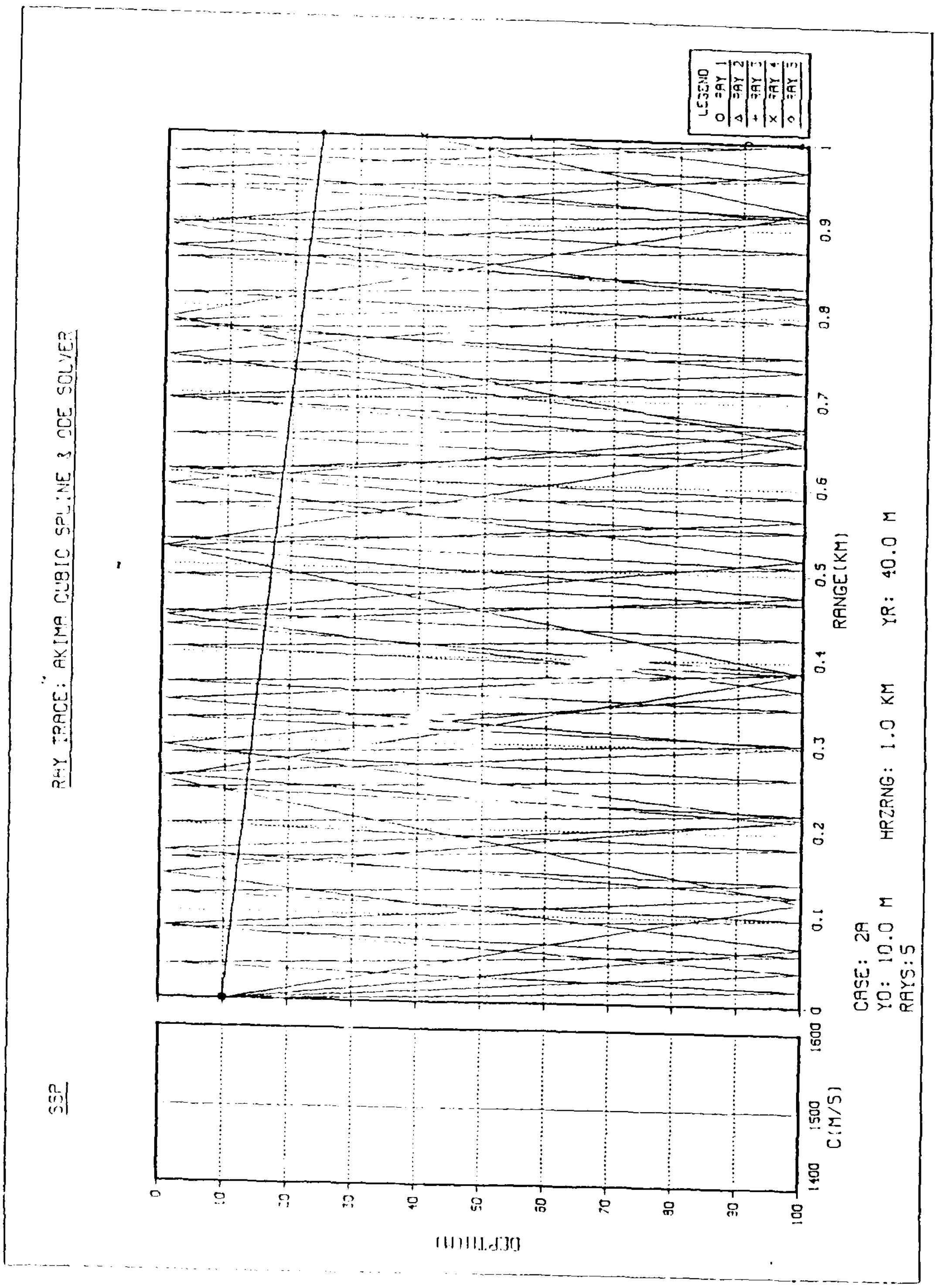

Figure 3.2 Ray trace correseponding to Table 3.5

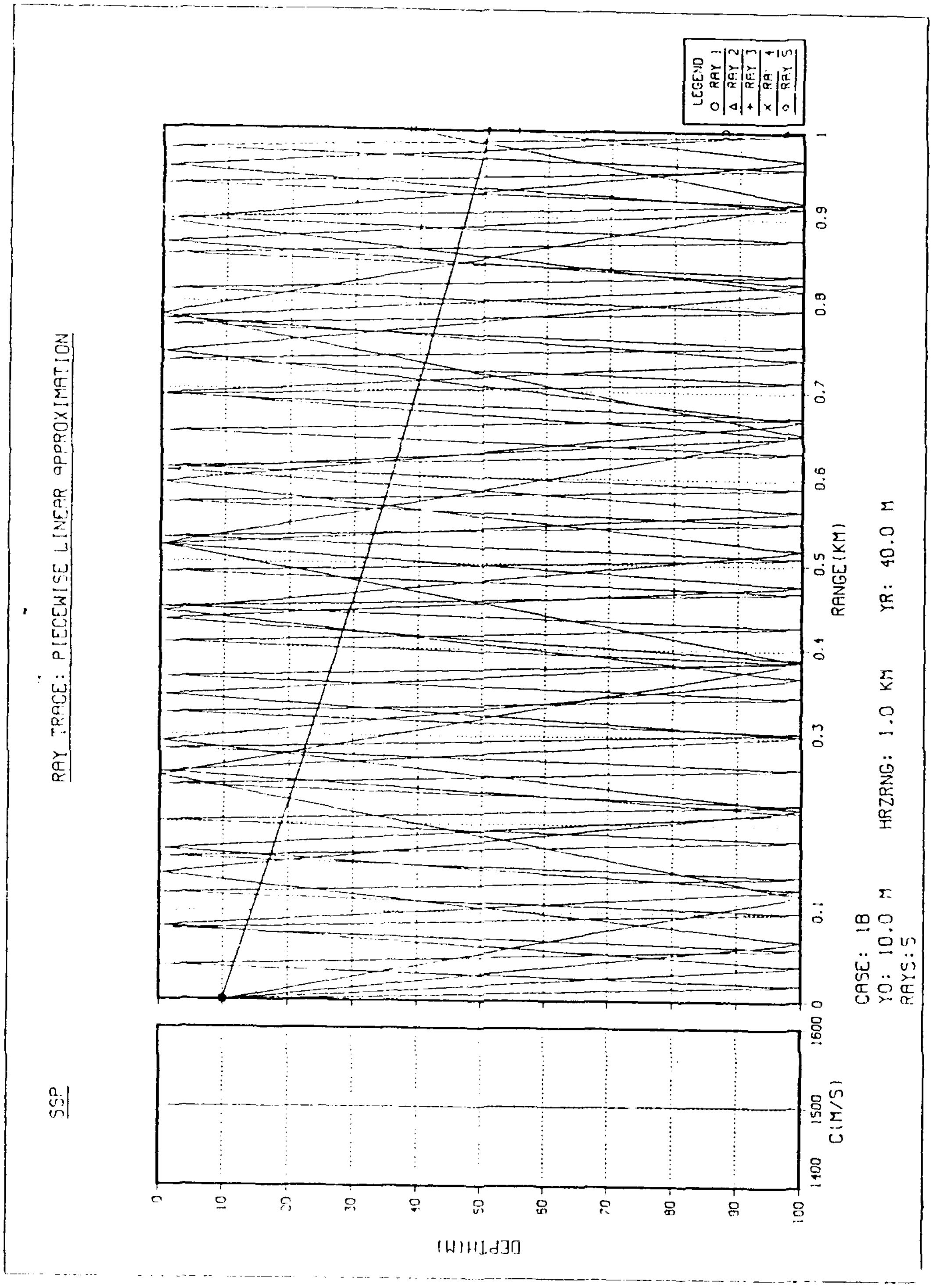

Figure 3.3 Ray trace corresponding to Table 3.6

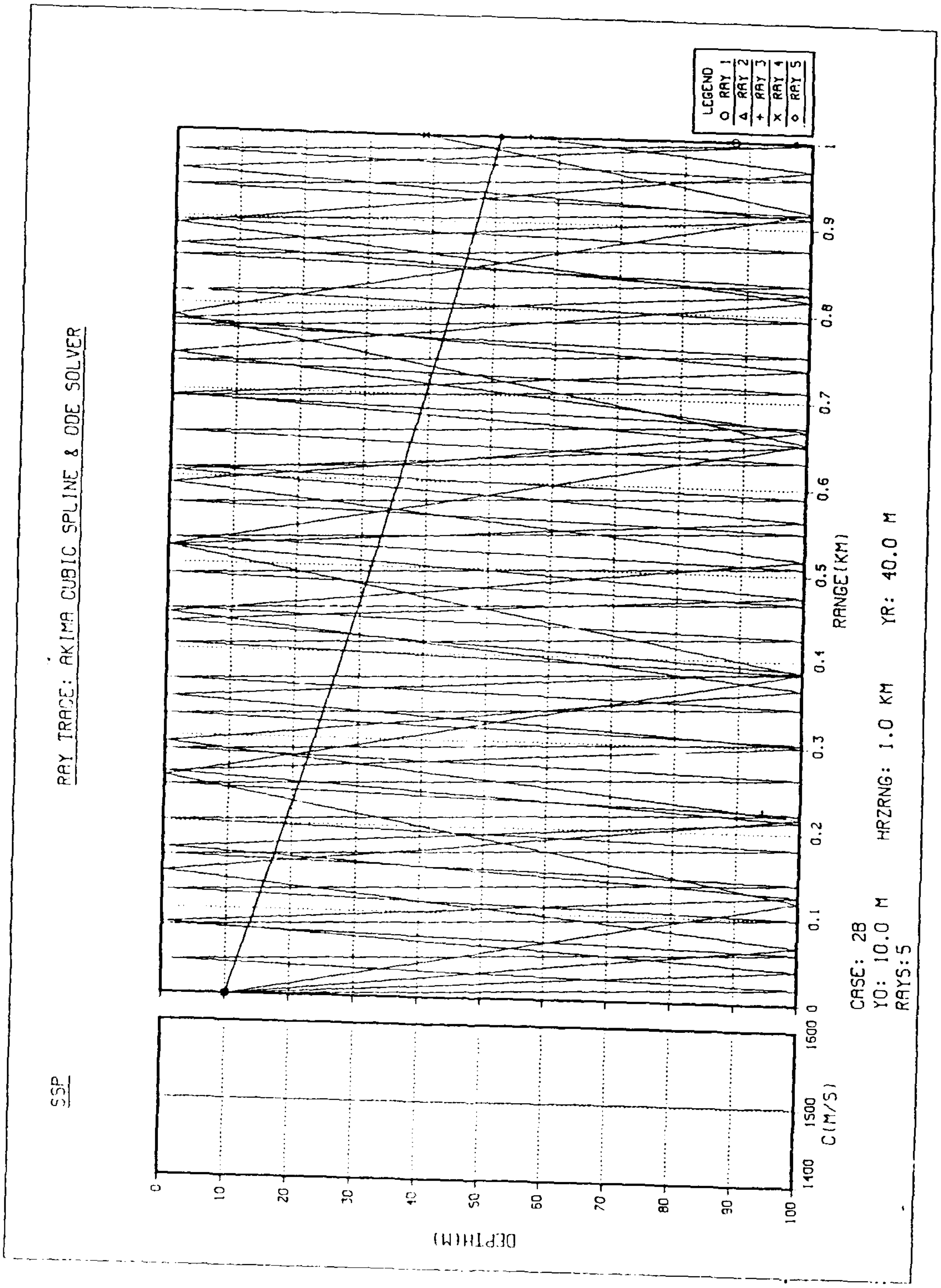

Figure 3.4 Ray trace corresponding to Table 3.7

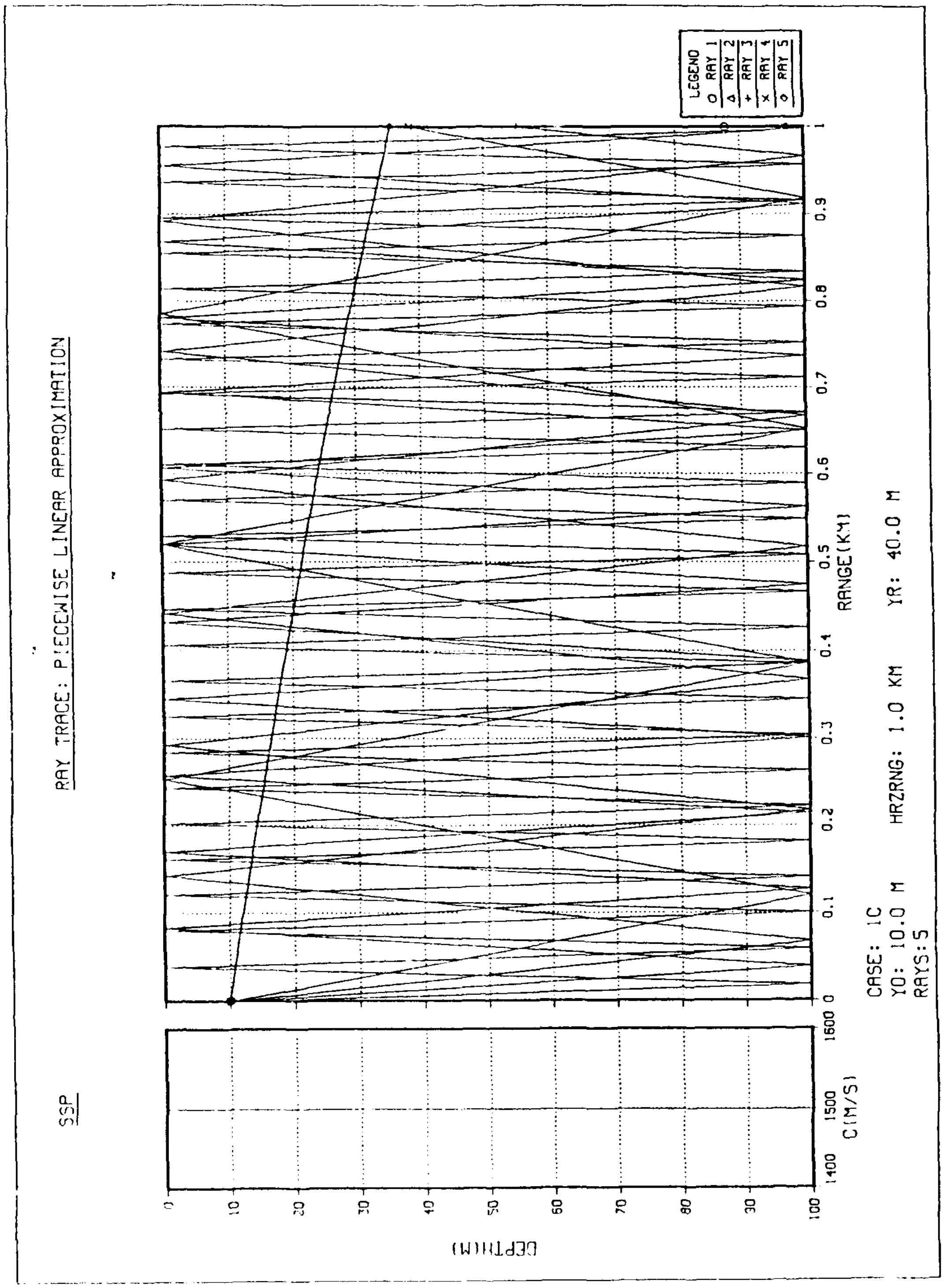

Figure 3.5 Ray trace corresponding to Table 3.8

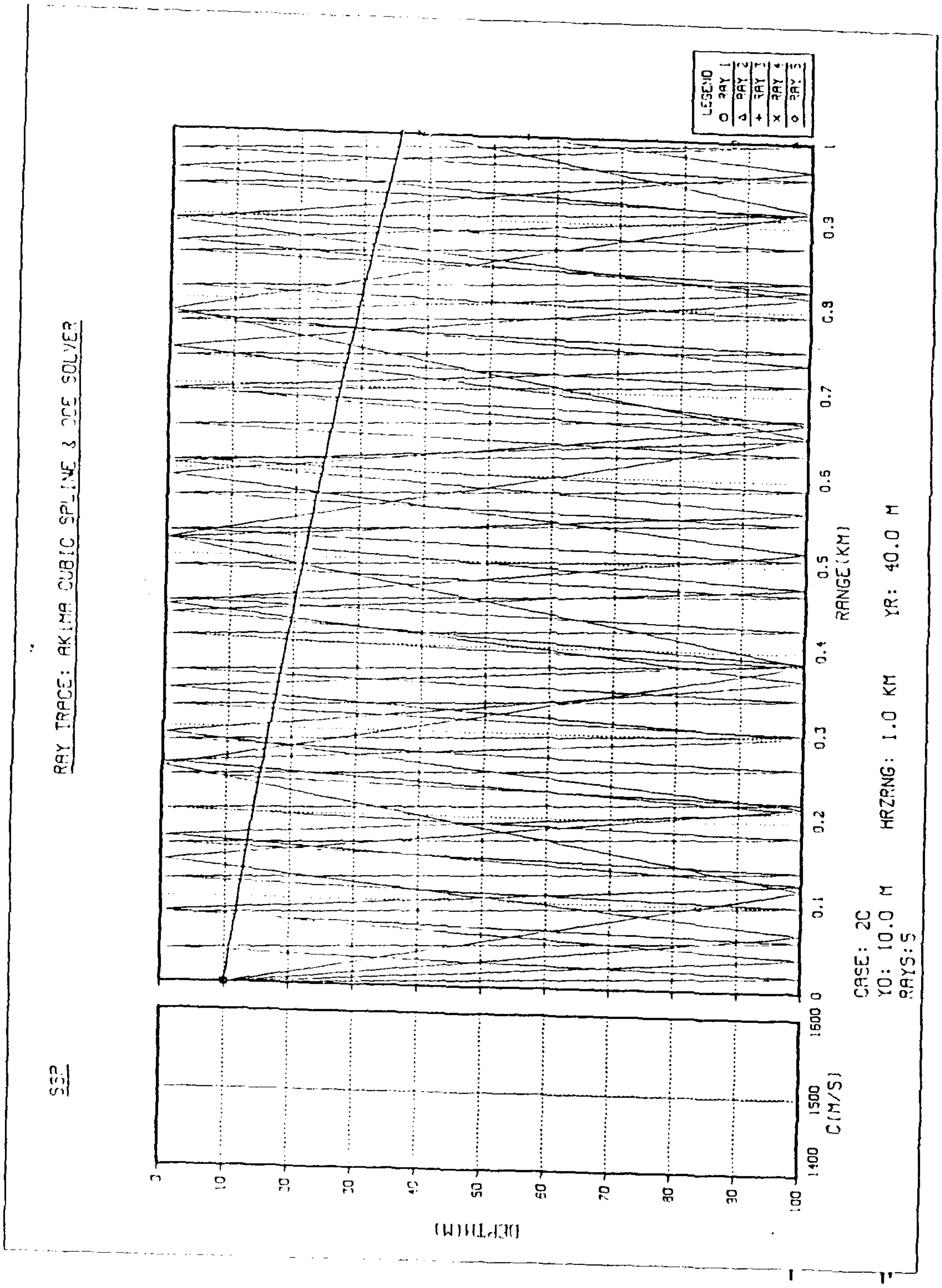

Figure 3.6 Ray trace corresponding to Table 3.9

results, no variations in depth or angle of arrival can be detected.

D. CHALLENGING TEST CASES

A smooth sound-speed profile in the form of a half sinusoid describes the medium for the next set of simulations. To this point, the agreement in accuracy between the two methods has been excellent; however, the shape of a sine curve is difficult to approximate with piecewise linear segments. While this will be a challenging test, smoothly curved profiles do occur naturally. The following set of values were used:

- 1500 m/sec is the sound speed at the ocean surface and at the 2000 meter bottom, and
- a minimum sound speed of 1475 m/sec occurs at the 1000 meter depth.

Three sets of simulations are presented using 5, 11, and 17 sound-speed versus depth data pairs. The simulation results will be examined to quantify the number of ocean medium sound-speed samples required by each method to converge to a solution. Also, the agreement in results for the two methods and the CPU times will be evaluated.

As the number of equally spaced data pairs varies, the medium and source conditions will be:

- source depth of 1000 meters,
- horizontal range of 3 kilometers, and
- five values of radial spatial frequency (FR) evaluated.

Tables 3.10, 3.11, and 3.12 show the results for the 5, 11, and 17 data pair test cases for the piecewise linear method. In comparing the first two of these tables, large discrepancies are seen in the final depths for each of the five radial frequencies evaluated—errors as large as seven meters. Likewise, travel time values differ by hundredths of a second meaning that modulo 2π phase values show no correlation between test cases. Unfortunately, the variations between Tables 3.11 and 3.12 where more data points are used, show the piecewise linear method is not converging to a solution.

Figures 3.7 through 3.9 correspond to Tables 3.10 thrcugh 3.12. While numerical phase values show gross disagreement, the inability to detect differences in the graphical representations illustrates the very sensitive nature of the phase calculations.

Tables 3.13 through 3.15 show the results when the ODE solver runs the simulations for 5, 11, and 17 data pairs. The 5 data pair case of Table 3.13 again shows significant disagreement with the Table 3.14 results using 11 data pairs. Final depths show differences of as much as a meter while travel times vary by milliseconds. As with the piecewise linear method, the modulo 2π phase values have not yet converged to a solution. Variations of up to 2 radians are seen.

TABLE 3.10
FIVE DATA PAIRS, PIECEWISE LINEAR SOLUTION

```
RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1S05

NDATA = 5                          NUMBER OF GRADIENTS = 4
G( 1) = -0.35355E-01 1/SEC   G( 2) = -0.14645E-01 1/SEC
G( 3) =  0.14645E-01 1/SEC   G( 4) =  0.35355E-01 1/SEC

ERAYS = F                          PRTALL = F
Y0 = 1000.0 M                      DEPTH = 2000.0 M    FREQC =     250.0 HZ       CMAX =  1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M      RATIO = 0.9999      FRMAX = 0.16665 CYCLES/M   NFRS =  5
DLTFR =  0.033330 CYCLES/M         RNGSTP =  1.0 M     HRZRNG =    3000.0 M       YR =   40.0 M
NOTE: FRMIN = DLTFR
```

NFR	BETAO(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	144.74	10.207186	168.504	0.1603341E+05	0.6282557E+03	0.1540515E+05	5.0674943
2	23.160	41.81	5.103311	156.449	0.8016262E+04	0.1256511E+04	0.6759751E+04	5.3264822
3	36.153	1064.45	3.402439	36.180	0.5344538E+04	0.1884767E+04	0.3459771E+04	4.0191220
4	51.868	682.47	2.554747	127.902	0.4012987E+04	0.2513023E+04	0.1499964E+04	4.5658515
5	79.494	1510.35	2.057830	81.249	0.3232433E+04	0.3141278E+04	0.9115404E+02	5.1894409

TOTAL CPU TIME = 0 MIN , 24.95 SEC

TABLE 3.11
ELEVEN DATA PAIRS, PIECEWISE LINEAR SOLUTION

```
RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1S11

NDATA = 11                         NUMBER OF GRADIENTS = 10
G( 1) = -0.38627E-01 1/SEC   G( 2) = -0.34846E-01 1/SEC
G( 3) = -0.27654E-01 1/SEC   G( 4) = -0.17755E-01 1/SEC
G( 5) = -0.61179E-02 1/SEC   G( 6) =  0.61179E-02 1/SEC
G( 7) =  0.17755E-01 1/SEC   G( 8) =  0.27654E-01 1/SEC
G( 9) =  0.34846E-01 1/SEC   G(10) =  0.38627E-01 1/SEC

ERAYS = F                          PRTALL = F
Y0 = 1000.0 M                      DEPTH = 2000.0 M    FREQC =     250.0 HZ       CMAX =  1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M      RATIO = 0.9999      FRMAX = 0.16665 CYCLES/M   NFRS =  5
DLTFR =  0.033330 CYCLES/M         RNGSTP =  1.0 M     HRZRNG =    3000.0 M       YR =   40.0 M
NOTE: FRMIN = DLTFR
```

NFR	BETAO(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	137.50	10.216764	168.506	0.1604846E+05	0.6282557E+03	0.1542020E+05	1.2628446
2	23.160	37.97	5.108073	156.449	0.8023742E+04	0.1256511E+04	0.6767231E+04	0.2400057
3	36.153	1067.39	3.405626	36.165	0.5349545E+04	0.1884767E+04	0.3464778E+04	2.7430820
4	51.868	679.33	2.557346	127.965	0.4017070E+04	0.2513023E+04	0.1504048E+04	2.3663691
5	79.494	1520.40	2.060168	81.354	0.3236104E+04	0.3141278E+04	0.9482580E+02	0.5780165

TOTAL CPU TIME = 0 MIN , 25.36 SEC

TABLE 3.12
SEVENTEEN DATA PAIRS, PIECEWISE LINEAR SOLUTION

```
RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1S17

NDATA = 17                      NUMBER OF GRADIENTS = 16
G( 1) = -0.39018E-01 1/SEC      G( 2) = -0.37519E-01 1/SEC
G( 3) = -0.34577E-01 1/SEC      G( 4) = -0.30307E-01 1/SEC
G( 5) = -0.24873E-01 1/SEC      G( 6) = -0.18482E-01 1/SEC
G( 7) = -0.11381E-01 1/SEC      G( 8) = -0.38429E-02 1/SEC
G( 9) =  0.38429E-02 1/SEC      G(10) =  0.11381E-01 1/SEC
G(11) =  0.18482E-01 1/SEC      G(12) =  0.24873E-01 1/SEC
G(13) =  0.30307E-01 1/SEC      G(14) =  0.34577E-01 1/SEC
G(15) =  0.37519E-01 1/SEC      G(16) =  0.39018E-01 1/SEC

ERAYS = F                          PRTALL = F
Y0 = 1000.0 M                      DEPTH = 2000.0 M    FREQC =     250.0 HZ       CMAX =  1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M      RATIO = 0.9999      FRMAX = 0.16665 CYCLES/M   NFRS =  5
DLTFR =  0.033330 CYCLES/M         RNGSTP =  1.0 M     HRZRNG =    3000.0 M       YR =   40.0 M
NOTE: FRMIN = DLTFR
```

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	136.67	10.217866	168.506	0.1605019E+05	0.6282557E+03	0.1542193E+05	2.9937577
2	23.160	37.53	5.108620	156.449	0.8024602E+04	0.1256511E+04	0.6768091E+04	1.1002238
3	36.153	1067.73	3.405996	36.160	0.5350126E+04	0.1884767E+04	0.3465359E+04	3.3242727
4	51.868	678.98	2.557644	127.973	0.4017538E+04	0.2513023E+04	0.1504515E+04	2.8338755
5	79.494	15[illegible]1.62	2.060452	81.322	0.3236550E+04	0.3141278E+04	0.9527150E+02	1.0237247

TOTAL CPU TIME = 0 MIN , 25.94 SEC

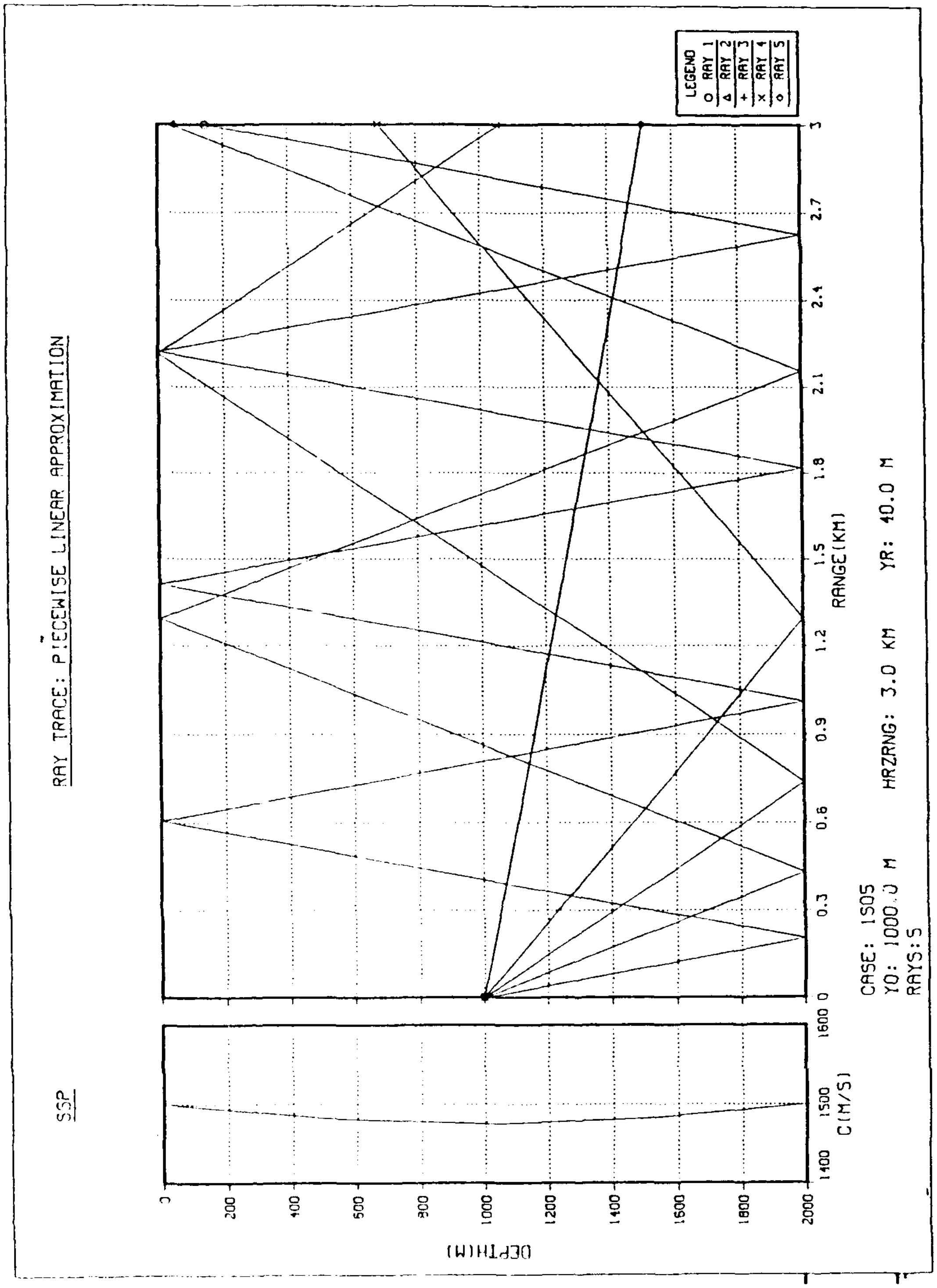

Figure 3.7 Ray trace corresponding to Table 3.10

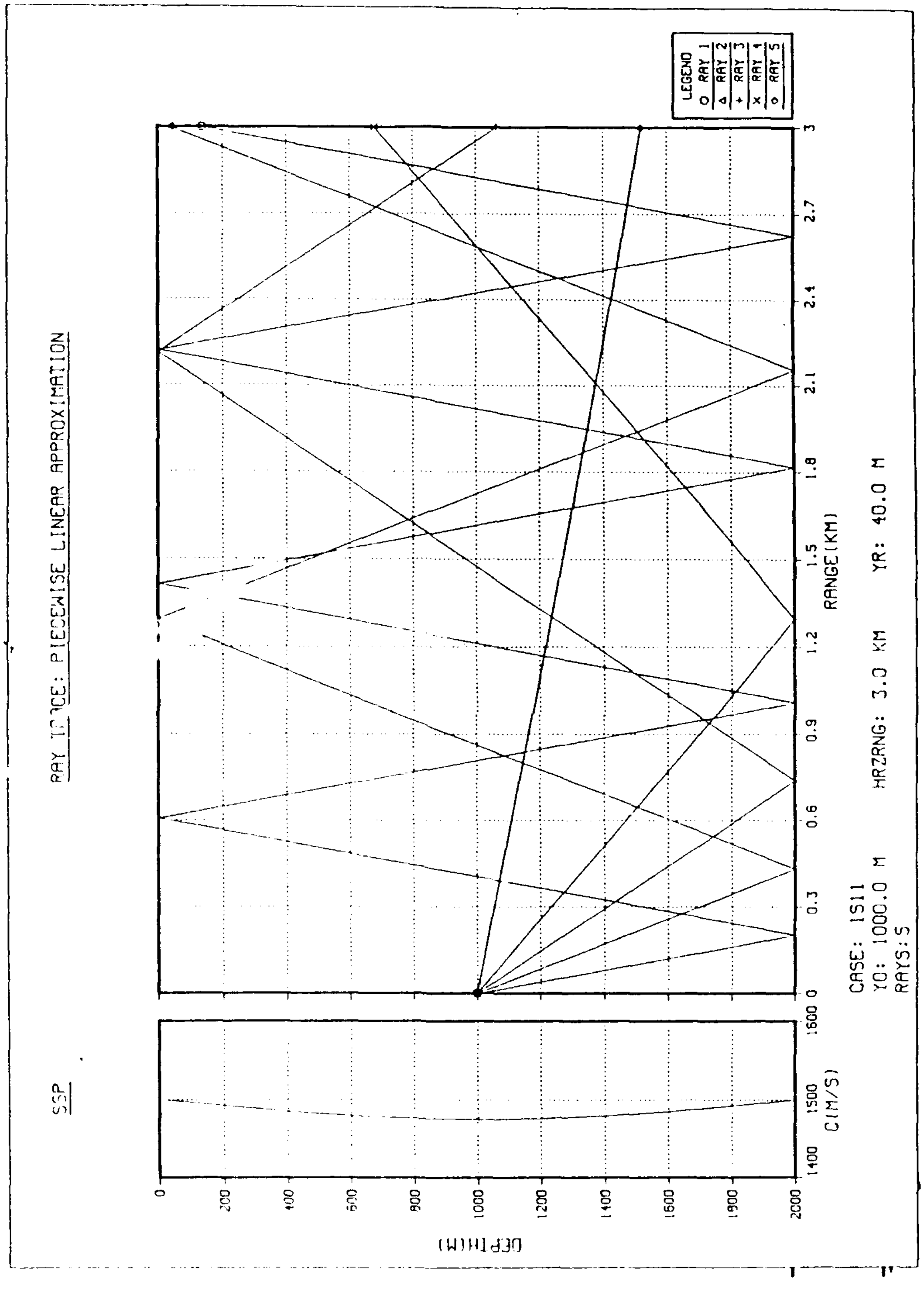

Figure 3.8 Ray trace corresponding to Table 3.11

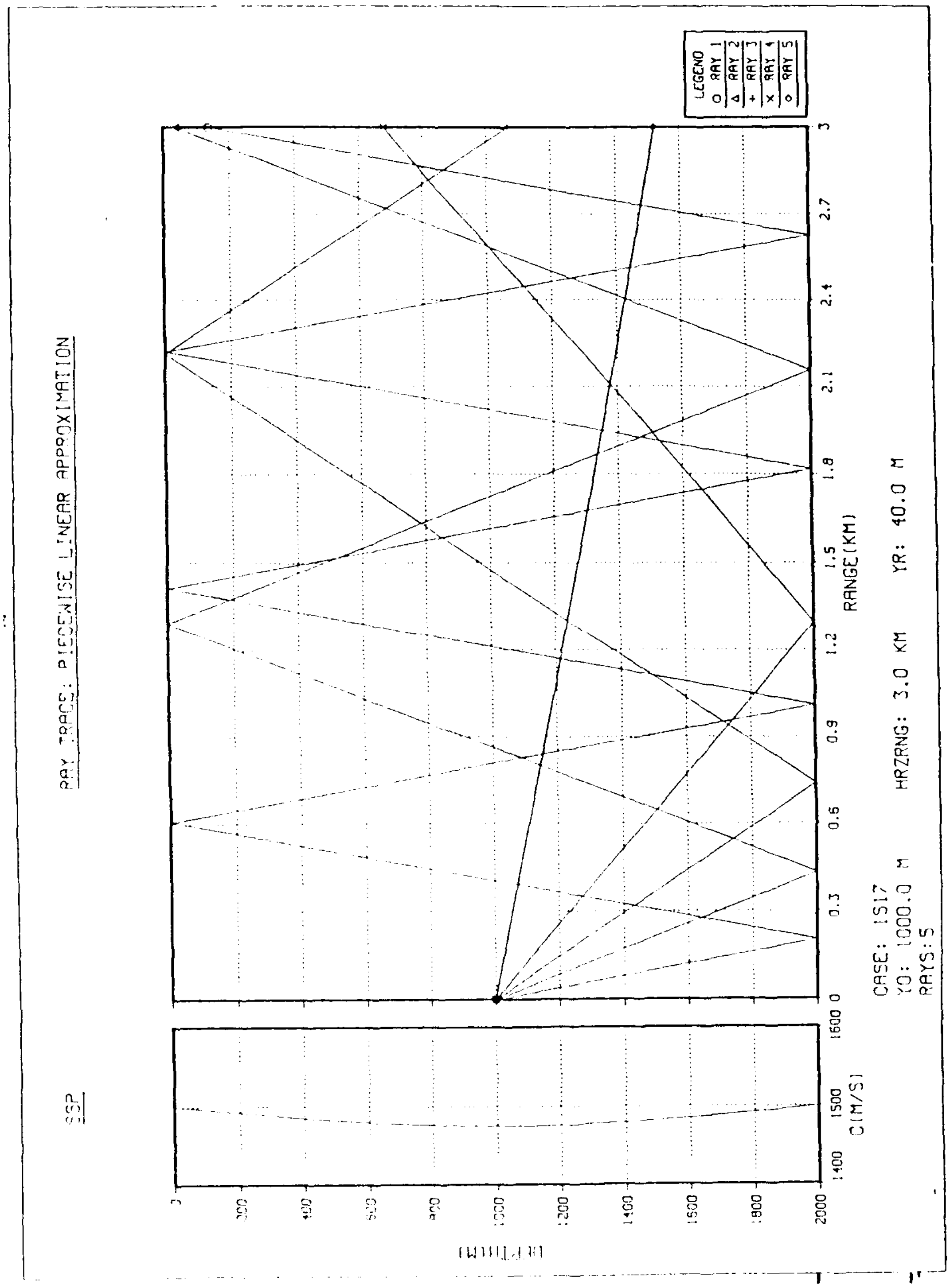

Figure 3.9 Ray trace corresponding to Table 3.12

TABLE 3.13
FIVE DATA PAIRS, ODE SOLVER

RAY TRACING USING AKIMA CUBIC SPLINE & ODE SOLVER
CASE 2S05

NDATA = 5

ERAYS = F		PRTALL = F					
Y0 = 1000.0 M		DEPTH = 2000.0 M		FREQC =	250.0 HZ	CMAX =	1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M		RATIO = 0.9999		FRMAX = 0.16665 CYCLES/M		NFRS =	5
DLTFR = 0.033330 CYCLES/M		RNGSTP = 1.0 M		HRZRNG =	3000.0 M	YR =	40.0 M

NOTE: FRMIN = DLTFR

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	136.85	10.220257	168.509	0.1605394E+05	0.6282557E+03	0.1542569E+05	0.4666177
2	23.160	36.56	5.109823	156.452	0.8026491E+04	0.1256511E+04	0.6769980E+04	2.9891864
3	36.153	1068.49	3.406808	36.157	0.5351402E+04	0.1884767E+04	0.3466635E+04	4.5997015
4	51.868	678.30	2.558194	127.973	0.4018402E+04	0.2513023E+04	0.1505379E+04	3.6978753
5	79.494	1521.82	2.060498	81.306	0.3236623E+04	0.3141278E+04	0.9534468E+02	1.0968988

TABLE 3.14
ELEVEN DATA PAIRS, ODE SOLVER

RAY TRACING USING AKIMA CUBIC SPLINE & ODE SOLVER
CASE 2S11

NDATA = 11

ERAYS = F		PRTALL = F					
Y0 = 1000.0 M		DEPTH = 2000.0 M		FREQC =	250.0 HZ	CMAX =	1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M		RATIO = 0.9999		FRMAX = 0.16665 CYCLES/M		NFRS =	5
DLTFR = 0.033330 CYCLES/M		RNGSTP = 1.0 M		HRZRNG =	3000.0 M	YR =	40.0 M

NOTE: FRMIN = DLTFR

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	136.09	10.218623	168.506	0.1605138E+05	0.6282557E+03	0.1542312E+05	4.1828982
2	23.160	37.23	5.108997	156.449	0.8025194E+04	0.1256511E+04	0.6768683E+04	1.6920625
3	36.153	1067.97	3.406254	36.157	0.5350531E+04	0.1884767E+04	0.3465764E+04	3.7288771
4	51.868	678.73	2.557848	127.978	0.4017858E+04	0.2513023E+04	0.1504835E+04	3.1540595
5	79.494	1522.39	2.060635	81.312	0.3236834E+04	0.3141278E+04	0.9555559E+02	1.3078080

TOTAL CPU TIME = 48 MIN , 14.53 SEC

TABLE 3.15
SEVENTEEN DATA PAIRS, ODE SOLVER

RAY TRACING USING AKIMA CUBIC SPLINE & ODE SOLVER
CASE 2S17

NDATA = 17

ERAYS = F		PRTALL = F					
Y0 = 1000.0 M		DEPTH = 2000.0 M		FREQC =	250.0 HZ	CMAX =	1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M		RATIO = 0.9999		FRMAX = 0.16665 CYCLES/M		NFRS =	5
DLTFR = 0.033330 CYCLES/M		RNGSTP = 1.0 M		HRZRNG =	3000.0 M	YR =	40.0 M

NOTE: FRMIN = DLTFR

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	136.13	10.218580	168.506	0.1605131E+05	0.6282557E+03	0.1542305E+05	4.1156874
2	23.160	37.25	5.108975	156.449	0.8025159E+04	0.1256511E+04	0.6768648E+04	1.6570617
3	36.153	1067.96	3.406239	36.157	0.5350507E+04	0.1884767E+04	0.3465740E+04	3.7048568
4	51.868	678.74	2.557836	127.978	0.4017839E+04	0.2513023E+04	0.1504817E+04	3.1353914
5	79.494	1522.36	2.060625	81.317	0.3236822E+04	0.3141278E+04	0.9554374E+02	1.2959589

TOTAL CPU TIME = 47 MIN , 45.20 SEC

A comparison of Tables 3.14 and 3.15 shows that phase calculation results converged for each of the five FR values. The increase in data pairs to 17 from 11 has changed the final depth values by only a few hundredths of a meter. Likewise, the very sensitive modulo 2π phase values show a change of only hundredths of a radian or less. The ray trace plots for Tables 3.13 through 3.15 are provided in Figures 3.10 through 3.12.

The CPU times for these three tables show the ODE solver to be costly, but insensitive to increases in the number of data pairs. That is, approximately the same amount of CPU time is required regardless of the number of data pairs used. The CPU time for the piecewise linear method also proved fairly insensitive to the number of data pairs used as seen in Tables 3.10 through 3.12.

The significant findings of this section are:

- the ODE solver can perform accurate phase calculations with only 17 data pairs sampled from a 2000 meter deep ocean having a curved, sinusoidal sound-speed profile,

- the piecewise linear approach does not converge to a solution using the 17 data pairs, and

- the differential equation solver is very costly to use in terms of CPU time versus the piecewise linear approach.

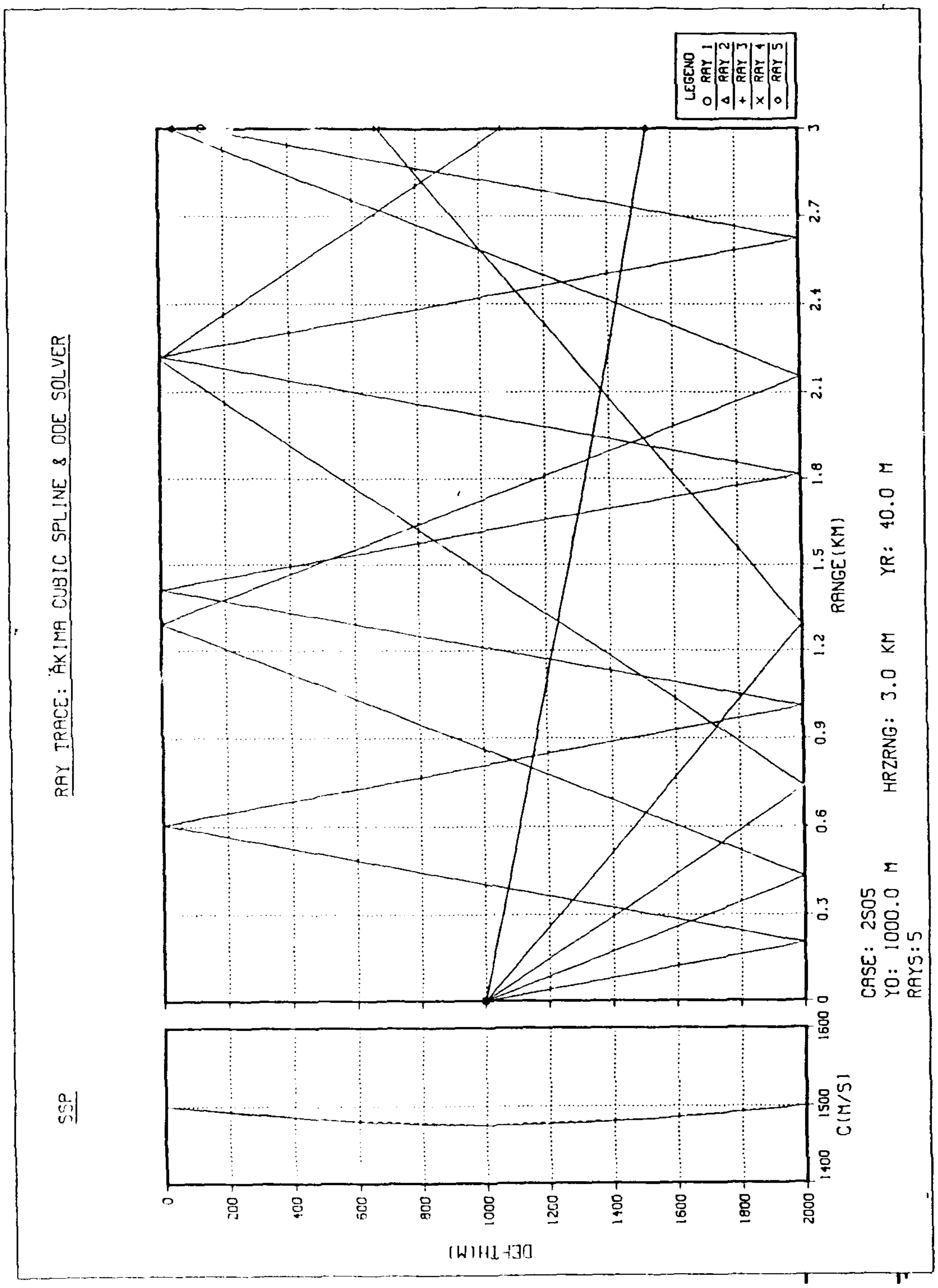

Figure 3.10 Ray trace corresponding to Table 3.13

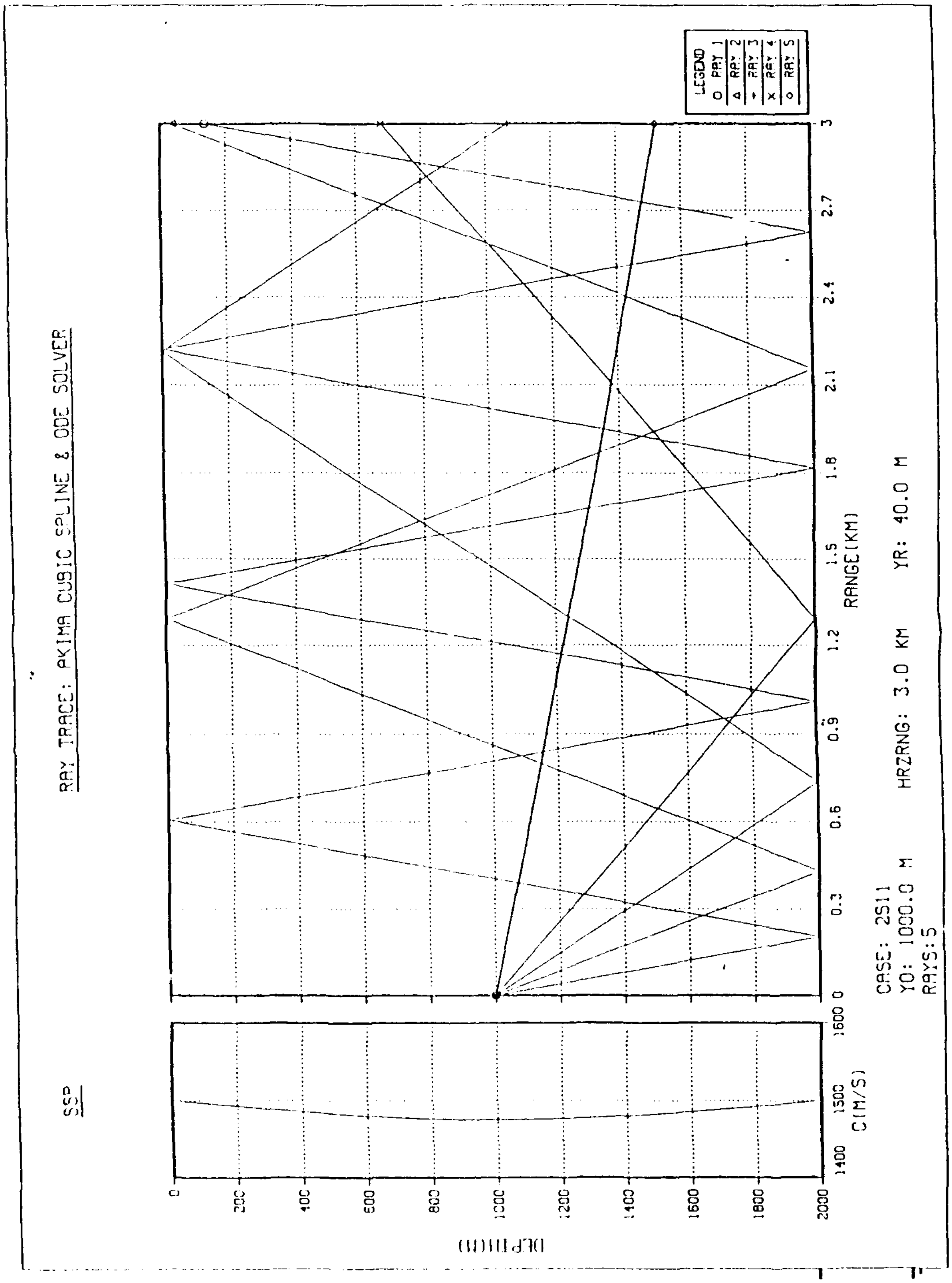

Figure 3.11 Ray trace corresponding to Table 3.14

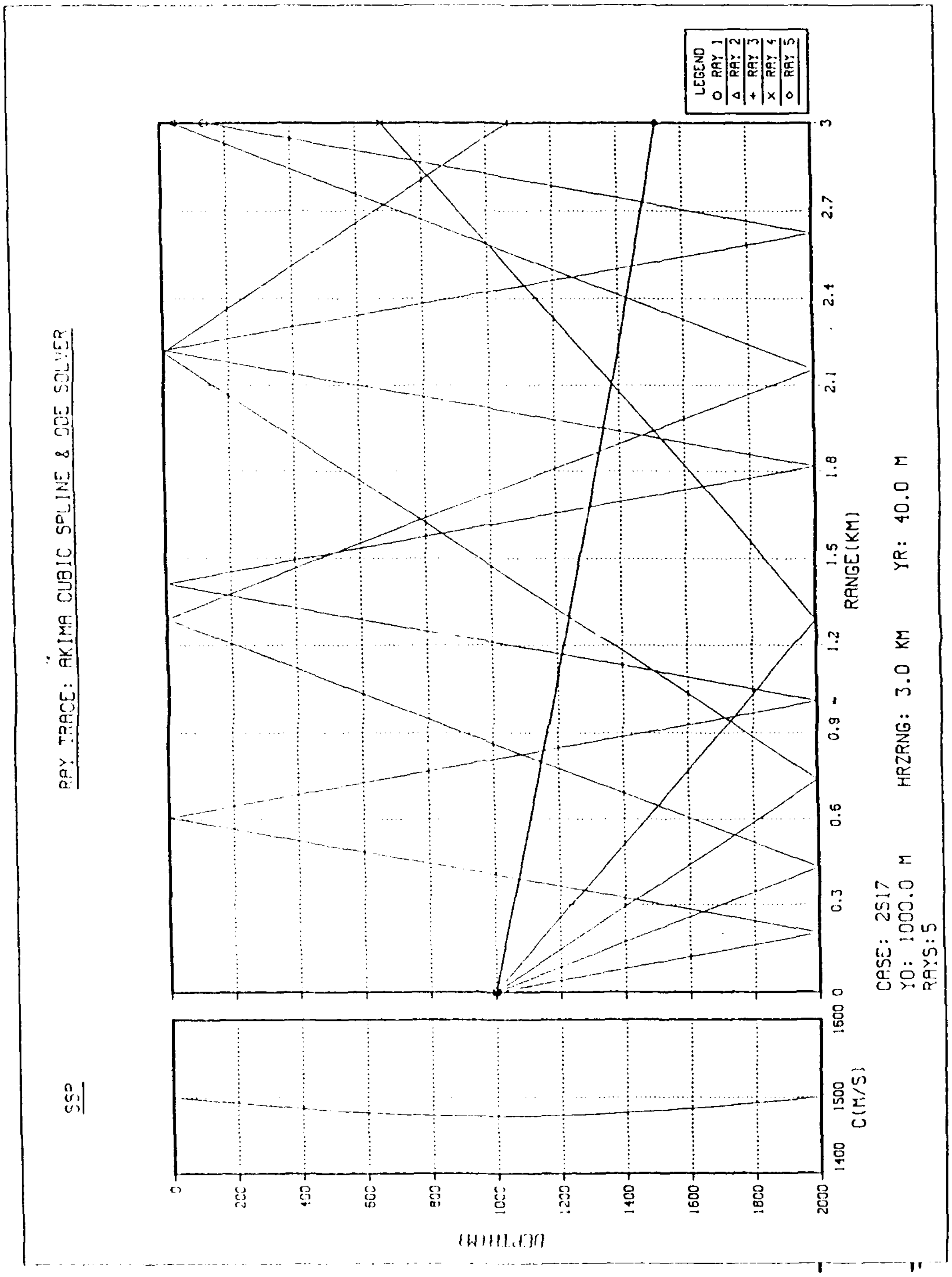

Figure 3.12 Ray trace corresponding to Table 3.15

E. SPECIAL INVESTIGATIONS

Each of the two phase calculation methods has shown one characteristic weakness. The piecewise linear method did not converge to a solution in the sinusoidal sound-speed profile case when 17 data pairs or samples were provided. In contrast, the ODE solver does converge to a solution, but is costly in terms of CPU time. This section will attempt to tailor the settings for each method to compensate for its weaknesses.

The simulation results in Tables 3.16 through 3.18 were produced by the piecewise linear method using 29, 55, and 65 data pairs, respectively. All medium and sound-speed profile characteristics remain the same as in the previous section. As the number of data pairs increases, the θ_y values are converging to the Table 3.15 solutions obtained with the ODE solver. Unfortunately, the phase values continue to show unacceptable errors of tenths and hundredths of a radian for the best case of 65 data pairs.

Table 3.19 shows the results for the ODE solver when the range step (RNGSTP) parameter is increased to five meters. As explained earlier, the range step is the differential equation system independent variable. All calculations prior to Table 3.19 used a conservative range step of one meter for both methods. Table 3.19 compares favorably with Table 3.15 achieving a balance between CPU time and accuracy. While the degradation in accuracy is only microseconds for travel time

TABLE 3.16

TWENTY-NINE DATA PAIRS, PIECEWISE LINEAR SOLUTION

```
RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1S29

NDATA = 29                        NUMBER OF GRADIENTS = 28
G( 1) = -0.39188E-01 1/SEC   G( 2) = -0.38695E-01 1/SEC
G( 3) = -0.37715E-01 1/SEC   G( 4) = -0.36262E-01 1/SEC
G( 5) = -0.34352E-01 1/SEC   G( 6) = -0.32010E-01 1/SEC
G( 7) = -0.29266E-01 1/SEC   G( 8) = -0.26154E-01 1/SEC
G( 9) = -0.22712E-01 1/SEC   G(10) = -0.18986E-01 1/SEC
G(11) = -0.15020E-01 1/SEC   G(12) = -0.10866E-01 1/SEC
G(13) = -0.65745E-02 1/SEC   G(14) = -0.22007E-02 1/SEC
G(15) =  0.22007E-02 1/SEC   G(16) =  0.65745E-02 1/SEC
G(17) =  0.10866E-01 1/SEC   G(18) =  0.15020E-01 1/SEC
G(19) =  0.18986E-01 1/SEC   G(20) =  0.22712E-01 1/SEC
G(21) =  0.26154E-01 1/SEC   G(22) =  0.29266E-01 1/SEC
G(23) =  0.32010E-01 1/SEC   G(24) =  0.34352E-01 1/SEC
G(25) =  0.36262E-01 1/SEC   G(26) =  0.37715E-01 1/SEC
G(27) =  0.38695E-01 1/SEC   G(28) =  0.39188E-01 1/SEC

ERAYS = F                        PRTALL = F
Y0 = 1000.0 M                    DEPTH = 2000.0 M    FREQC =     250.0 HZ       CMAX =  1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M    RATIO = 0.9999      FRMAX = 0.16665 CYCLES/M   NFRS =  5
DLTFR =  0.033330 CYCLES/M       RNGSTP =  1.0 M     HRZRNG =    3000.0 M       YR =   40.0 M
NOTE: FRMIN = DLTFR
```

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	136.31	10.218342	168.506	0.1605093E+05	0.6282557E+03	0.1542268E+05	3.7413092
2	23.160	37.34	5.108857	156.449	0.8024973E+04	0.1256511E+04	0.6768462E+04	1.4710844
3	36.153	1067.88	3.406158	36.157	0.5350380E+04	0.1884767E+04	0.3465613E+04	3.5775983
4	51.868	678.82	2.557772	127.976	0.4017739E+04	0.2513023E+04	0.1504716E+04	3.0348743
5	79.494	1522.11	2.060567	81.320	0.3236730E+04	0.3141278E+04	0.9545197E+02	1.2041934

TOTAL CPU TIME = 0 MIN , 26.46 SEC

TABLE 3.17
FIFTY-FIVE DATA PAIRS, PIECEWISE LINEAR SOLUTION

RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1S55

NDATA = 55 NUMBER OF GRADIENTS = 54

G(1) = -0.39248E-01 1/SEC	G(2) = -0.39115E-01 1/SEC
G(3) = -0.38850E-01 1/SEC	G(4) = -0.38453E-01 1/SEC
G(5) = -0.37926E-01 1/SEC	G(6) = -0.37271E-01 1/SEC
G(7) = -0.36490E-01 1/SEC	G(8) = -0.35586E-01 1/SEC
G(9) = -0.34561E-01 1/SEC	G(10) = -0.33419E-01 1/SEC
G(11) = -0.32163E-01 1/SEC	G(12) = -0.30800E-01 1/SEC
G(13) = -0.29331E-01 1/SEC	G(14) = -0.27764E-01 1/SEC
G(15) = -0.26103E-01 1/SEC	G(16) = -0.24353E-01 1/SEC
G(17) = -0.22521E-01 1/SEC	G(18) = -0.20613E-01 1/SEC
G(19) = -0.18635E-01 1/SEC	G(20) = -0.16594E-01 1/SEC
G(21) = -0.14497E-01 1/SEC	G(22) = -0.12350E-01 1/SEC
G(23) = -0.10162E-01 1/SEC	G(24) = -0.79399E-02 1/SEC
G(25) = -0.56907E-02 1/SEC	G(26) = -0.34221E-02 1/SEC
G(27) = -0.11420E-02 1/SEC	G(28) = 0.11420E-02 1/SEC
G(29) = 0.34221E-02 1/SEC	G(30) = 0.56907E-02 1/SEC
G(31) = 0.79399E-02 1/SEC	G(32) = 0.10162E-01 1/SEC
G(33) = 0.12350E-01 1/SEC	G(34) = 0.14497E-01 1/SEC
G(35) = 0.16594E-01 1/SEC	G(36) = 0.18635E-01 1/SEC
G(37) = 0.20613E-01 1/SEC	G(38) = 0.22521E-01 1/SEC
G(39) = 0.24353E-01 1/SEC	G(40) = 0.26103E-01 1/SEC
G(41) = 0.27764E-01 1/SEC	G(42) = 0.29331E-01 1/SEC
G(43) = 0.30800E-01 1/SEC	G(44) = 0.32163E-01 1/SEC
G(45) = 0.33419E-01 1/SEC	G(46) = 0.34561E-01 1/SEC
G(47) = 0.35586E-01 1/SEC	G(48) = 0.36490E-01 1/SEC
G(49) = 0.37271E-01 1/SEC	G(50) = 0.37926E-01 1/SEC
G(51) = 0.38453E-01 1/SEC	G(52) = 0.38850E-01 1/SEC
G(53) = 0.39115E-01 1/SEC	G(54) = 0.39248E-01 1/SEC

ERAYS = F PRTALL = F
Y0 = 1000.0 M DEPTH = 2000.0 M FREQC = 250.0 HZ CMAX = 1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M RATIO = 0.9999 FRMAX = 0.16665 CYCLES/M NFRS = 5
DLTFR = 0.033330 CYCLES/M RNGSTP = 1.0 M HRZRNG = 3000.0 M YR = 40.0 M
NOTE: FRMIN = DLTFR

NFR	BETAO(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	136.18	10.218510	168.506	0.1605120E+05	0.6282557E+03	0.1542294E+05	4.0060532
2	23.160	37.28	5.108940	156.449	0.8025104E+04	0.1256511E+04	0.6768593E+04	1.6024783
3	36.153	1067.94	3.406215	36.157	0.5350470E+04	0.1884767E+04	0.3465702E+04.	3.6673828
4	51.868	678.77	2.557817	127.977	0.4017810E+04	0.2513023E+04	0.1504787E+04	3.1058334
5	79.494	1522.29	2.060608	81.317	0.3236795E+04	0.3141278E+04	0.9551684E+02	1.2690645

TOTAL CPU TIME = 0 MIN , 26.61 SEC

TABLE 3.18
SIXTY-FIVE DATA PAIRS, PIECEWISE LINEAR SOLUTION

```
RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1S65

NDATA = 65                        NUMBER OF GRADIENTS = 64
G( 1) = -0.39254E-01 1/SEC        G( 2) = -0.39160E-01 1/SEC
G( 3) = -0.38971E-01 1/SEC        G( 4) = -0.38688E-01 1/SEC
G( 5) = -0.38312E-01 1/SEC        G( 6) = -0.37844E-01 1/SEC
G( 7) = -0.37284E-01 1/SEC        G( 8) = -0.36635E-01 1/SEC
G( 9) = -0.35897E-01 1/SEC        G(10) = -0.35073E-01 1/SEC
G(11) = -0.34165E-01 1/SEC        G(12) = -0.33174E-01 1/SEC
G(13) = -0.32103E-01 1/SEC        G(14) = -0.30955E-01 1/SEC
G(15) = -0.29733E-01 1/SEC        G(16) = -0.28438E-01 1/SEC
G(17) = -0.27075E-01 1/SEC        G(18) = -0.25647E-01 1/SEC
G(19) = -0.24158E-01 1/SEC        G(20) = -0.22610E-01 1/SEC
G(21) = -0.21007E-01 1/SEC        G(22) = -0.19354E-01 1/SEC
G(23) = -0.17654E-01 1/SEC        G(24) = -0.15912E-01 1/SEC
G(25) = -0.14132E-01 1/SEC        G(26) = -0.12317E-01 1/SEC
G(27) = -0.10473E-01 1/SEC        G(28) = -0.86032E-02 1/SEC
G(29) = -0.67130E-02 1/SEC        G(30) = -0.48066E-02 1/SEC
G(31) = -0.28886E-02 1/SEC        G(32) = -0.96364E-03 1/SEC
G(33) =  0.96364E-03 1/SEC        G(34) =  0.28886E-02 1/SEC
G(35) =  0.48066E-02 1/SEC        G(36) =  0.67130E-02 1/SEC
G(37) =  0.86032E-02 1/SEC        G(38) =  0.10473E-01 1/SEC
G(39) =  0.12317E-01 1/SEC        G(40) =  0.14132E-01 1/SEC
G(41) =  0.15912E-01 1/SEC        G(42) =  0.17654E-01 1/SEC
G(43) =  0.19354E-01 1/SEC        G(44) =  0.21007E-01 1/SEC
G(45) =  0.22610E-01 1/SEC        G(46) =  0.24158E-01 1/SEC
G(47) =  0.25647E-01 1/SEC        G(48) =  0.27075E-01 1/SEC
G(49) =  0.28438E-01 1/SEC        G(50) =  0.29733E-01 1/SEC
G(51) =  0.30955E-01 1/SEC        G(52) =  0.32103E-01 1/SEC
G(53) =  0.33174E-01 1/SEC        G(54) =  0.34165E-01 1/SEC
G(55) =  0.35073E-01 1/SEC        G(56) =  0.35897E-01 1/SEC
G(57) =  0.36635E-01 1/SEC        G(58) =  0.37284E-01 1/SEC
G(59) =  0.37844E-01 1/SEC        G(60) =  0.38312E-01 1/SEC
G(61) =  0.38688E-01 1/SEC        G(62) =  0.38971E-01 1/SEC
G(63) =  0.39160E-01 1/SEC        G(64) =  0.39254E-01 1/SEC

ERAYS = F                         PRTALL = F
Y0 = 1000.0 M                     DEPTH = 2000.0 M   FREQC =    250.0 HZ        CMAX =  1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M     RATIO = 0.9999     FRMAX = 0.16665 CYCLES/M  NFRS =  5
DLTFR =  0.033330 CYCLES/M        RNGSTP =  1.0 M    HRZRNG =   3000.0 M       YR =   40.0 M
NOTE: FRMIN = DLTFR
```

NFR	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAV(DEG)	THETAT(RAD)	THETAR(RAD)	THETAV(RAD)	MODULO TWOPI THETAV(RAD)
1	11.341	136.17	10.218528	168.506	0.1605123E+05	0.6282557E+03	0.1542297E+05	4.0340889
2	23.160	37.27	5.108949	156.449	0.8025118E+04	0.1256511E+04	0.6768607E+04	1.6163983
3	36.153	1067.94	3.406221	36.157	0.5350479E+04	0.1884767E+04	0.3465712E+04	3.6769515
4	51.868	678.76	2.557822	127.978	0.4017818E+04	0.2513023E+04	0.1504795E+04	3.1134251
5	79.494	1522.31	2.060612	81.317	0.3236802E+04	0.3141278E+04	0.9552360E+02	1.2758237

```
TOTAL CPU TIME = 0 MIN , 26.84 SEC
```

TABLE 3.19
INCREASED RANGE STEP, ODE SOLVER SOLUTION

```
RAY TRACING USING AKIMA CUBIC SPLINE & ODE SOLVER
CASE 2S17

NDATA = 17

ERAYS = F                       PRTALL = F
Y0 = 1000.0 M                   DEPTH = 2000.0 M    FREQC =     250.0 HZ          CMAX =  1500.0 M/SEC
FREQC/CMAX = 0.16667 CYCLES/M   RATIO = 0.9999      FRMAX = 0.16665 CYCLES/M  NFRS =  5
DLTFR =  0.033330 CYCLES/M      RNGSTP =  5.0 M     HRZRNG =    3000.0 M      YR =   40.0 M
NOTE: FRMIN = DLTFR
```

NFR	BETAO(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	11.341	136.09	10.218584	168.506	0.1605131E+05	0.6282557E+03	0.1542306E+05	4.1219846
2	23.160	37.25	5.108975	156.449	0.8025159E+04	0.1256511E+04	0.6768648E+04	1.6570457
3	36.153	1067.96	3.406239	36.157	0.5350507E+04	0.1884767E+04	0.3465740E+04	3.7047770
4	51.868	678.74	2.557836	127.978	0.4017839E+04	0.2513023E+04	0.1504817E+04	3.1353500
5	79.494	1522.36	2.060625	81.317	0.3236822E+04	0.3141278E+04	0.9554379E+02	1.2960153

TOTAL CPU TIME = 10 MIN , 3.53 SEC

and hundreds or thousandths of a radian for modulo 2π θ_Y, the reduction in CPU time is following a nearly linear relation to the increase in range step, that is, CPU time was reduced approximately by one-fifth.

F. EIGENRAYS

Tables 3.20 and 3.21 show the eigenrays found by the two ray acoustics methods. Figures 3.13 and 3.14 correspond to these two tables. The figures show the sound-speed profile to be that of a SOFAR channel. This channel profile was chosen because it produces interesting ray patterns, and it commonly occurs in nature. The minimum sound-speed occurs at 1000 meters depth and a local minima occurs at the ocean surface. The tables show that each method found the same eigenrays at

TABLE 3.20
EIGENRAYS FOUND USING THE
PIECEWISE LINEAR APPROXIMATION METHOD

```
RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1CH4

NDATA =  4                    NUMBER OF GRADIENTS =  3
G( 1) =  0.16000E-01 1/SEC   G( 2) = -0.18889E-01 1/SEC
G( 3) =  0.17000E-01 1/SEC

ERAYS = T                     PRTALL = F
Y0 =  100.0 M                 DEPTH = 2000.0 M        FREQC =     250.0 HZ
RNGSTP =    5.0 M             ANGSTP =    1.0 DEG     HRZRNG =     200.0 M      YR =  250.0 M
YERROR =    5.0 M
```

EIGENRAYS

RAY	BETAO(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	53.000	251.11	0.167269	52.856	0.2627462E+03	0.1672491E+03	0.9549710E+02	1.2493187
2	54.000	245.69	0.165112	53.856	0.2593569E+03	0.1694232E+03	0.8993371E+02	1.9691119
3	150.000	246.73	0.267040	29.939	0.4194649E+03	0.1047093E+03	0.3147556E+03	0.5963779

TOTAL CPU TIME = 0 MIN , 22.47 SEC

TABLE 3.21
EIGENRAYS FOUND USING THE
ODE SOLVER METHOD

```
RAY TRACING USING AKIMA CUBIC SPLINE & ODE SOLVER
CASE 2CH4

NDATA =  4

ERAYS = T                     PRTALL = F
Y0 =  100.0 M                 DEPTH = 2000.0 M        FREQC =     250.0 HZ
RNGSTP =    5.0 M             ANGSTP =    1.0 DEG     HRZRNG =     200.0 M      YR =  250.0 M
YERROR =    5.0 M
```

EIGENRAYS

RAY	BETAO(DEG)	DEPTH(M)	TRVLT(SEC)	BETAY(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	53.000	250.85	0.167063	52.930	0.2626214E+03	0.1672491E+03	0.9517250E+02	0.9247239
2	54.000	245.44	0.164912	53.932	0.2590435E+03	0.1694232E+03	0.8962032E+02	1.6557224
3	150.000	246.56	0.264843	29.971	0.4191563E+03	0.1047093E+03	0.3144470E+03	0.2877732

TOTAL CPU TIME = 24 MIN , 12.94 SEC

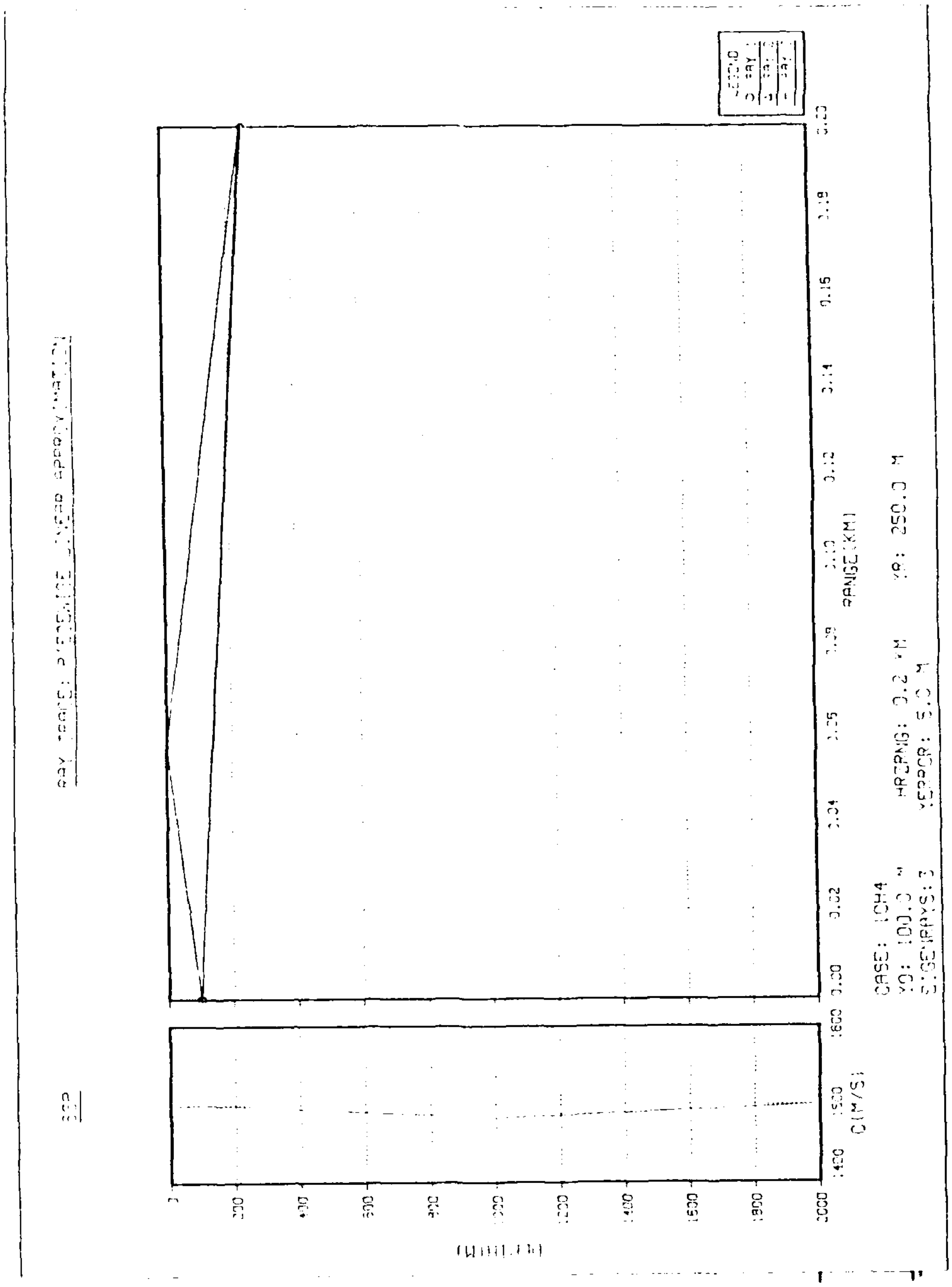

Figure 3.13 Ray trace plot corresponding to Table 3.20

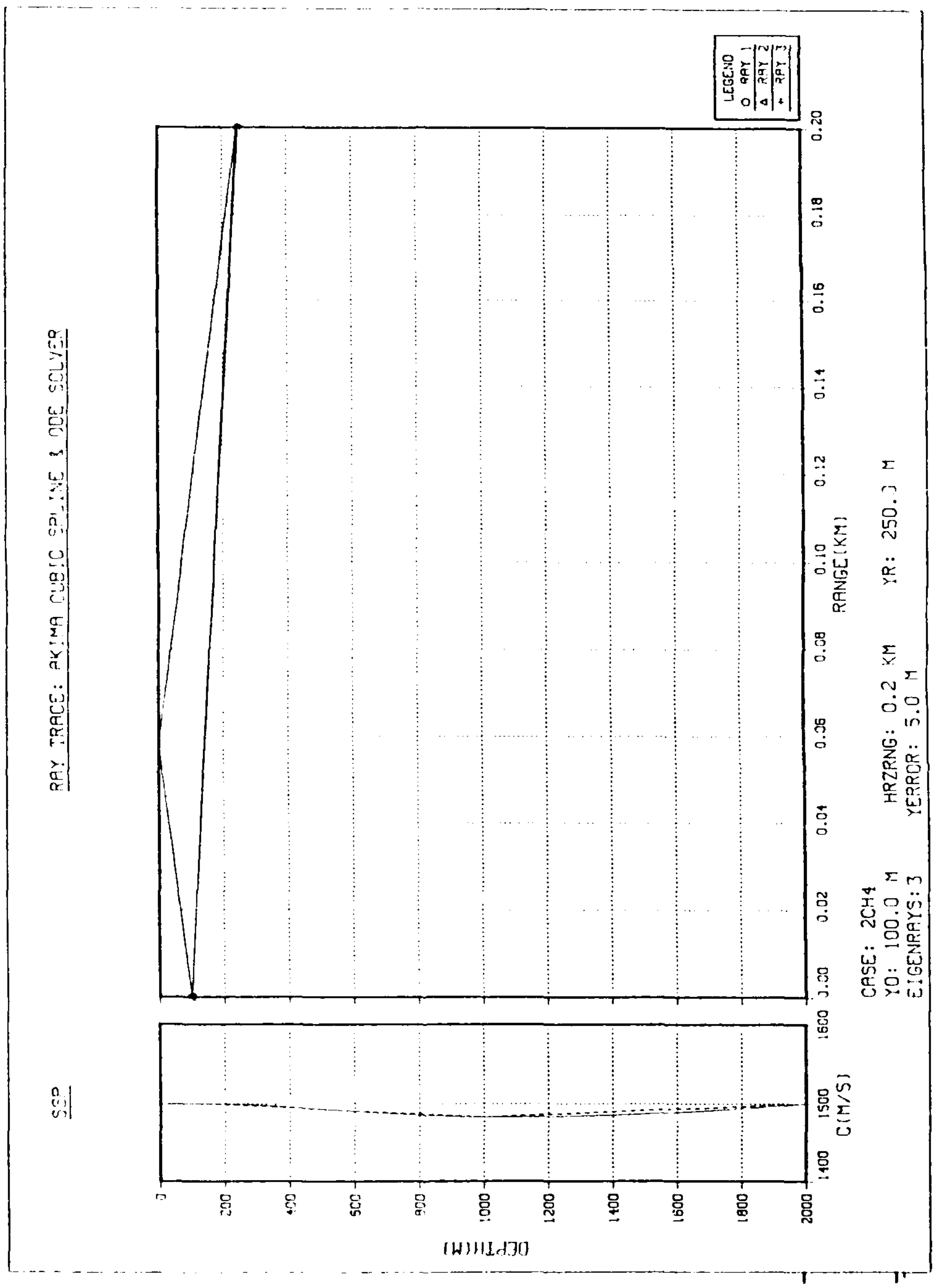

Figure 3.14 Ray trace plot corresponding to Table 3.21

launch angles of 53, 54 and 150 degrees. The tables highlight this problem's parameters to be:

- a source depth y_0 of 100 meters;
- a receiver depth y_r of 250 meters;
- and a horizontal range (HRZRNG) to the receiver of 200 meters.

As expected, the ODE solver runs into range limitations imposed by CPU time requirements. The 200 meter example presented consumed over 24 minutes of CPU time using the ODE solver versus the 21 seconds used by the piecewise linear method.

Tables 3.22 and 3.23 (along with their corresponding Figures 3.15 and 3.16) show solutions to long-range eigenray search problems. These two examples continue to use a source depth y_0 of 100 meters and a receiver depth y_r of 250 meters. Table 3.22 shows a horizontal range (HRZRNG) of 10 kilometers with a depth error y_{error} at the receiver of 2 meters. The angle step size (angstp) between each ray is 0.1°. Table 3.22 also shows that three eigenrays were found with launch angles of 51.2°, 90.9° and 129.5°. The CPU time for this 10km run is shown to be 8.5 minutes.

TABLE 3.22
LONG RANGE EIGENRAYS, PIECEWISE LINEAR SOLUTION

RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1CH4

NDATA = 4 NUMBER OF GRADIENTS = 3
G(1) = 0.16000E-01 1/SEC G(2) = -0.18889E-01 1/SEC
G(3) = 0.17000E-01 1/SEC

ERAYS = T PRTALL = F
Y0 = 100.0 M DEPTH = 2000.0 M FREQC = 250.0 HZ
RNGSTP = 5.0 M ANGSTP = 0.1 DEG HRZRNG = 10000.0 M VR = 250.0 M
YERROR = 2.0 M

EIGENRAYS

RAY	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAV(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	51.200	248.91	8.645958	51.067	0.1358104E+05	0.8160392E+04	0.5420648E+04	4.5420569
2	90.900	250.77	6.674996	86.356	0.1048506E+05	0.1046966E+05	0.1542213E+02	2.8557634
3	129.500	249.21	8.730378	50.370	0.1371365E+05	0.8079626E+04	0.5634020E+04	4.2855981

TOTAL CPU TIME = 8 MIN , 31.38 SEC

TABLE 3.23
LONG RANGE EIGENRAYS, PIECEWISE LINEAR SOLUTION

RAY TRACING USING PIECEWISE LINEAR APPROXIMATION
CASE 1CH4

NDATA = 4 NUMBER OF GRADIENTS = 3
G(1) = 0.16000E-01 1/SEC G(2) = -0.18889E-01 1/SEC
G(3) = 0.17000E-01 1/SEC

ERAYS = T PRTALL = F
Y0 = 100.0 M DEPTH = 2000.0 M FREQC = 250.0 HZ
RNGSTP = 5.0 M ANGSTP = 1.0 DEG HRZRNG = 50000.0 M VR = 250.0 M
YERROR = 15.0 M

EIGENRAYS

RAY	BETA0(DEG)	DEPTH(M)	TRVLT(SEC)	BETAV(DEG)	THETAT(RAD)	THETAR(RAD)	THETAY(RAD)	MODULO TWOPI THETAY(RAD)
1	40.000	236.69	52.421823	39.917	0.8234401E+05	0.3365291E+05	0.4869109E+05	2.6894315
2	102.000	261.00	34.440292	102.535	0.5409868E+05	0.5121057E+05	0.2888117E+04	4.1350528
3	119.000	238.33	38.520812	60.820	0.6050835E+05	0.4579040E+05	0.1471795E+05	2.7282732
4	122.000	257.74	39.734666	122.182	0.6241507E+05	0.4439925E+05	0.1801581E+05	1.9203099

TOTAL CPU TIME = 4 MIN , 27.61 SEC

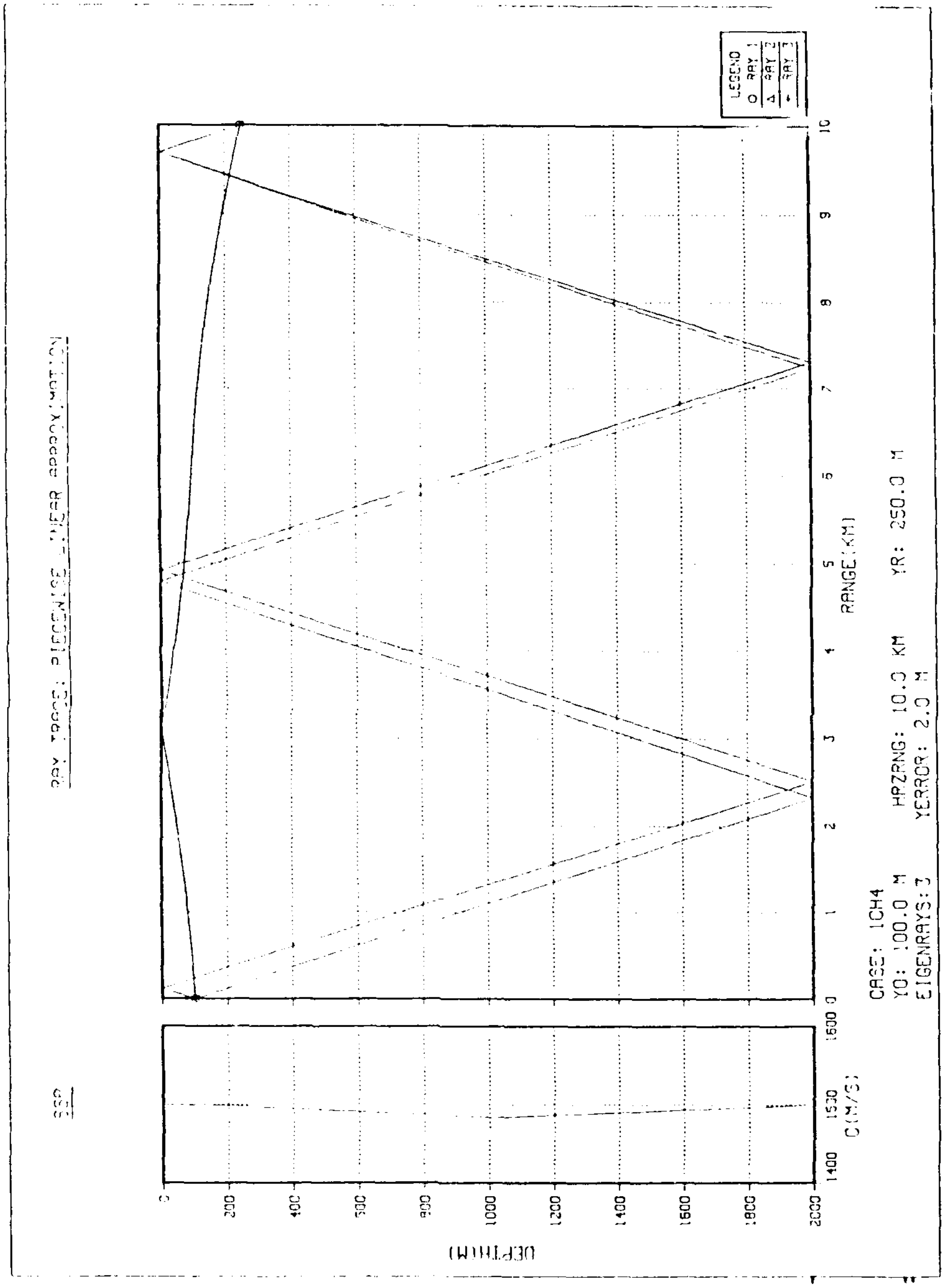

Figure 3.15 Ray trace plot corresponding to Table 3.22

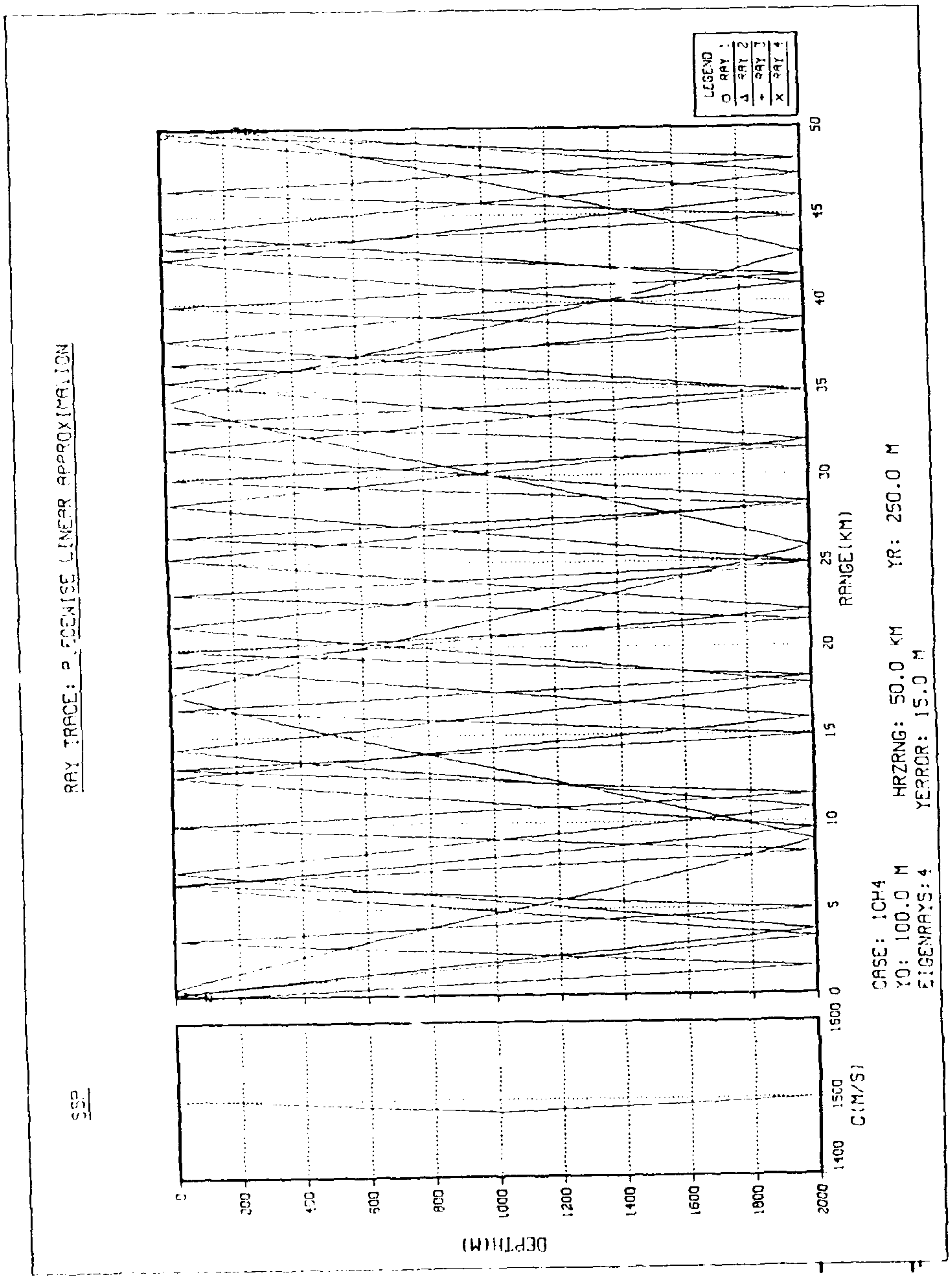

Figure 3.16 Ray trace plot corresponding to Table 3.23

Table 3.23 shows simulation results using the piecewise linear method with a horizontal range (HRZRNG) of 50km. Parameters that affect execution time such as angle step size, depths, etc., match the simulation presented in Table 3.21 using the ODE solver. Table 3.21 shows that the ODE solver used 24 minutes to complete a 200 meter range eigenvalue problem. In comparison, Table 3.23 shows that the piecewise linear method completed the 50km run in only 4 minutes.

In summary, this chapter has shown that

- the ODE solver performs accurate phase calculations with far fewer data pair samples when a smooth sound-speed profile is encountered;

- increasing the range step size of the ODE solver to five meters reduces the CPU time cost without signficantly affecting the phase solution accuracies;

- the piecewise linear method can run long-range simulations using relatively little CPU time; and

- the piecewise linear method requires many sound-speed data pair samples for an accurate solution in a medium with a smooth, sinusoidal profile.

IV. CONCLUSIONS AND RECOMMENDATIONS

The transfer function of the inhomogeneous ocean based on the WKB approximation requires solving a phase integral. Ray acoustics theory can provide solutions to this phase integral. The simulations performed show that the travel times calculated using the theories of ray acoustics can be used to solve the phase integral avoiding direct numerical integration.

Two applications of ray acoustics produced computer simulation codes which

- are capable of solving for the position, travel time and phase of a propagating ray, and
- have very different advantages and costs.

The first application was the piecewise linear approximation. Sound-speed versus depth data pairs sampled from the ocean medium were connected with constant gradient linear segments. Well-known, closed form equations form the mathematical model for sound propagation. The solutions are low cost (in terms of CPU time), but many data samples are required for accurate phase solutions for arbitrary sound-speed profiles.

In contrast, the Akima cubic spline/ODE solver method uses the medium samples to form a continuously variable sound-speed profile. Accurate phase calculations can be made with a minimum of sound-speed versus depth data pairs. The disadvantage in using the ODE solver is its exorbitant cost in terms of CPU time. Simulations must be short range problems to limit computing costs.

The ability to search for and identify eigenrays was developed for each of the two ray acoustics theory applications. Very distinctive characteristics were seen for phase calculations in terms of the number of data samples needed to assure accurate solutions. The identification of eigenrays is an easier task of position or depth computation. Here the distinctions tend to disappear with the two methods arriving at the same solutions for eigenray launch angles. The cost or CPU time required continues to strongly favor using the piecewise linear approach.

These findings indicate that most propagation problems will require a piecewise linear approach for computational efficiency. Using the ODE solver would severely limit the range of computer simulations.

Careful sampling of the ocean media is required to obtain accurate results from the piecewise linear method. Sufficient sound-speed versus depth data pairs must be used to accurately represent the sound-speed profile.

This thesis developed the tools to quantify the strengths and weaknesses of two phase computation methods in a variety of media. With these findings in mind, future work recommendations are to

- incorporate each phase computation technique as a module in the larger pulse propagation code; and

- run pulse propagation simulations to compare the received pulse shapes.

If this comparison shows that the piecewise linear approximation produces a relatively undistorted received pulse, it is an efficient solution to the phase computation problem.

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